Gas refractive index measurements using dynamic phase-shift keyed gratings

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We present a new technique for measuring refractive index changes using the technique of phase-shift keying of holographic gratings. Based on this technique, we have realized an holographic interferometer to gauge refractive index variations in gases. We present experimental data for standard atmosphere under laboratory conditions and theoretical calculations for the measured transfer function of phase-shifted Bragg gratings.

1 Experimental setup and working principle

We present a new type of interferometer, that is based on the phase-shift keying technique [1]. The interferometer is used to gauge small refractive index variations in an air filled cuvette due to small pressure changes. The interferometer employs two different geometries for the recording and the readout process (cf. fig. 1). The recording of the holographic gratings is done in transmission geometry, using a signal 9b and a reference 9a beam which overlap with a third recording beam 9c and subsequently form two adjoining holographic gratings (6a and 6b) in a BaTiO3 crystal. Hereby, the signal beam 9b is guided through a cuvette (5) filled with gas and later rejoined with the reference beam. Both beams overlap with the recording beam and form a phase-shift keyed grating in the medium via the photorefractive effect [2]. The phase-step in the combined wavefront of both beams and the resulting gap in the combined grating is a function of the pressure in the cuvette.

This combined grating can be analyzed in real-time using an additional IR beam as a probe-beam in reflection geometry. This probe-beam is diffracted from both gratings and the two reconstructed beams are coherently superimposed. Tuning the IR-laser yields the spectral depending diffraction efficiency of the combined grating - the transfer function [1].

\[ \eta(\lambda) = \frac{l_{\text{out}}}{l_{\text{in}}} \]  

(1)

Here, \( l_{\text{in}} \) and \( l_{\text{out}} \) are the incoming and reflected portions of the IR-probebeam. \( l_{\text{out}} \) is the coherent superposition of two reflected portions, from both gratings, respectively. The shape of the transfer function is a function of the gap \( \Delta \lambda \) between the two gratings.

![Fig. 2 Recording geometry](image)

The grating gap and thus the phase shift \( \Delta \phi \) between the signal and the reference beam is determined by fitting a numerically simulated function to the measured transfer function. The fitting parameters of the calculated transfer function yield the phase shift between the signal and the reference...
beam. For a given length $l$, the refractive index variation $\Delta n$ can be calculated:

$$\Delta n = \frac{\varphi}{2\pi \lambda} \times \frac{\lambda}{l} \quad (2)$$

### 2 Experimental results

We investigated the refractive index variation due to a pressure change for an atmospheric gas composition. During our experiments, the temperature of the cuvette was kept constant at $T = 294 \pm 0.5 K$. The pressure in the cuvette was simultaneously monitored in order to validate the experimental results. In fig.3 one can see the analysis of the measured transfer function. The dotted line represents experimental data (no interpolated data) and the superimposed solid line shows the theoretically calculated transfer function. The latter was fitted to the experimental results. In this case, the analysis yielded a phase shift of $135^\circ$. In fig.4 one can see the calculated refractive index variation based on five transfer functions. The associated pressure was measured using a barometer, which is also the source of the given error (for the pressure). The error in the phase-measurement is about $1/40\pi$. The solid line was calculated using the ideal gas equation to validate the experimental results.

$$\Delta n = (n_0 - 1) \times \left( \frac{\rho}{T} \right) \times \left( \frac{T_0}{\rho_0} \right) \quad (3)$$

Evidently the actual phase-measurement is too precise to be validated by the used barometer.

### 3 Numerical simulations

The used numerical simulations of the transfer functions for phase-shift keyed gratings were calculated using the fourier transform of the phase-profile of the used recording wavefront.

$$\eta(\lambda) \propto F[\Delta n(z) \exp[i\varphi(z)]] \quad (4)$$

Here, $F$ denotes the Fourier transform, $z$ the propagation axis of the IR-beam, $\eta$ is the diffraction efficiency (the transfer function), $\Delta n$ and $\varphi$ are the spatial varying grating amplitude and phase, respectively. This method is only valid for the case of low diffraction efficiency ($\eta < 10\%$), for higher diffraction efficiencies, simulations based on the fourier transform strongly deviate from the experimental results.

### 4 Conclusions

The employed combination of two different geometries for the analysis of the fringe patterns in the holographic medium yields a high signal-to-noise ratio of the measurement signal and therefore a high resolution for the interferometer. At the present state, the adaptive holographic interferometer has a resolution of $1/40\pi$. This considerable high resolution is due to the high signal-to-noise ratio of the reflected IR beam. Furthermore, the quantization of the measured phase-shifts can be performed within this resolution limit due to an excellent agreement between the theoretical simulations and the measured transfer functions. The use of a dynamic holographic media for the recording and the evaluation process of the Bragg gratings effectively stabilizes the modified Mach-Zehnder interferometer (cf. fig.1) in respect to parasitic phase changes. The necessary theoretical simulations for the phase-retrieval from the measured transfer functions were done using a previously developed method based on Fourier transformation [1] which incorporates the necessary phase shift between two gratings as well as variations of the grating amplitude. The presented measurement method was used to measure refractive index changes in gases with a resolution of $\Delta n = 1.3 \times 10^{-7}$.

![Fig. 1 Refractive index measurement](image-url)

**Fig. 1** Refractive index measurement

### References
