Light concentration efficiency of diffractive lenses with overlapping apertures

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Diffractive lenses with overlapping apertures enable a dense spot-wise illumination with high aperture foci. A relative light concentration efficiency is defined to investigate the effect of the overlapping apertures and potential post-processing steps compared to an ideal focus. It is found that particularly for diffractive lenses with overlapping apertures and high numerical aperture the post-processing results only in a constant multiplicative loss.

1 Introduction

Diffractive lenses with overlapping apertures enable a dense spot-wise illumination with high aperture foci, which is not achievable with traditional refractive lens arrays. Particularly the numerical aperture (NA) of a diffractive lens with overlapping apertures is a free design parameter and independent of the focal length and the focal pitch of the lens array. This dense focal grid allows a high degree of parallelization which can be used in applications like scanning microscopy or direct laser writing.

2 Mathematical description of diffractive lenses with overlapping apertures

The diffractive lens array with overlapping apertures is a periodic optical component in thin element approximation which can be expressed by a Fourier series

$$u_{DOE}\left(r_{\perp}\right)=\frac{1}{P^{2}}\sum_{l,m}\tilde{t}_{l,m}^{DOE}e^{i\frac{2\pi}{P}(xl+ym)}. \qquad (1)$$

The complex amplitude is given by the Fourier reconstruction of the Fourier coefficients

$$\tilde{t}_{lm}^{DOE}=\iint \mathrm{circ}\left(\tfrac{s_{\perp}}{NA}\right)e^{-ikf\sqrt{1-s_{\perp}^{2}}}\delta\left(s_{x}-\tfrac{l\lambda}{P},s_{y}-\tfrac{m\lambda}{P}\right)d^{2}\mathbf{s}_{\perp} \qquad (2)$$

which are generated from a discretized homogenous pupil with an additional phase factor which accounts for the propagation distance f between the focal and the element plane. The frequency discretization is the reason for the periodicity and enables the overlapping of the apertures. In the extreme case $P\rightarrow\infty$ of a continuous frequency space, the description of the diffractive optical element reduces to an ideal focusing wave front [1]. For this ideal case, the focal intensity $I_{ideal}\left(0,f\right)$ and the total power P_{ideal} of the wave can be calculated analytically.

3 Light concentration

To evaluate the light concentration of a single focus, the generalized Strehl criteria [2] is a common tool which is independent of scaling:

$$\eta=\frac{I\left(0,f\right)}{SP} \qquad (3)$$

Here, the focal intensity $I\left(0,f\right)$ is compared to the power P of the wave and the area of the entrance pupil S in frequency space is used for normalization. This results in a generalized Strehl value of unity for the ideal focusing wave front which is the basis for the lenses with overlapping apertures.

4 Relative light concentration efficiency

To evaluate the light concentration in a focal spot, the focal intensity $I\left(0,f\right)$ produced by a lens array with overlapping apertures is compared against the focal intensity $I_{ideal}\left(0,f\right)$ of the ideal wave with no frequency discretization. Therefore the relative light concentration efficiency is defined as

$$S_{\mathrm{mod}}=\frac{I\left(0,f\right)}{I_{ideal}\left(0,f\right)}\frac{P_{ideal}}{P}. \qquad (4)$$

The power of the waves is used for normalization. Due to the overlap, the power per period is used for the lens with overlapping apertures.

This definition of the relative light concentration efficiency allows to assess the influences of the overlap onto the light concentration as well as the influences from potential post-processing. In particular the effect of phase discretization and amplitude leveling which are common in the fabrication of diffractive optical elements can be evaluated.

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5 Effect of phase quantization

In Fig. 1, the effect of phase quantization onto the relative light concentration efficiency is plotted for lens arrays with fixed focal pitch and focal length but different numerical apertures.

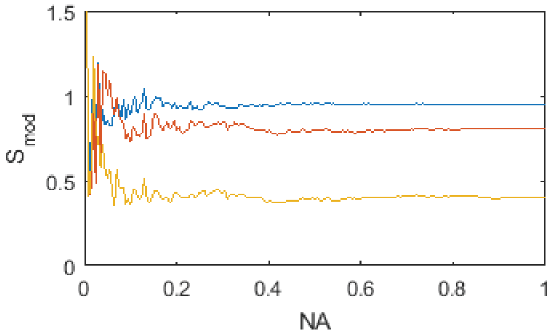

Fig. 1 *Relative light concentration efficiency for lens arrays with continuous amplitude and 2 (orange), 4 (red) and 8 (blue) discrete phase levels.*

For high numerical apertures, the phase quantization results in a multiplicative loss. The amount agrees well with the upper diffraction efficiency bound given by [2]:

$$\eta = \frac{N^2}{\pi^2} \sin^2\left(\frac{\pi}{N}\right) \tag{5}$$

Here, N is the number of discrete phase levels. For low numerical apertures we see deviations from this constant.

6 Effect of amplitude leveling

In Fig. 2, the effect of sole amplitude leveling and amplitude leveling combined with phase binarization is shown. For comparison, the relative light concentration efficiency for arrays with continuous amplitude and with and without phase binarization is also shown.

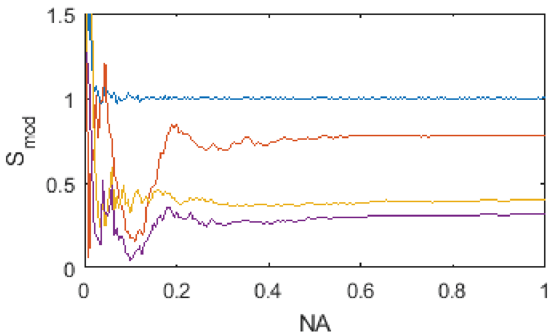

Fig. 2 *Relative light concentration efficiency for elements with continuous amplitude and phase (blue), leveled amplitude and continuous phase (red), continuous amplitude and binarized phase (orange) and leveled amplitude and binarized phase (violet).*

Again, for high numerical apertures, the amplitude leveling results in a multiplicative loss. The effects from phase quantization and amplitude levelling seem to be independent. The loss factor in the case of a leveled amplitude and binarized phase is the product of the loss factors of each individual effect. For lower numerical apertures, a serve deviation from a constant loss factor is found, the reason for this is explained in the next section.

7 Focal modulation for low numerical apertures

For low numerical apertures the amplitude leveling results in a modulation of the focal profile, an example for NA = 0.1 is shown in Fig. 3.

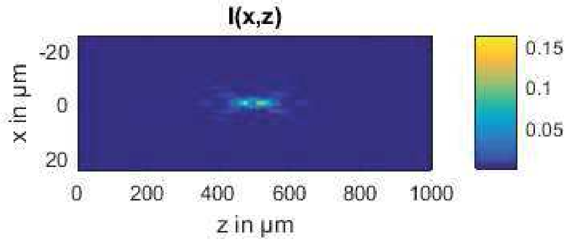

Fig. 3 *Focal profile of an amplitude leveled diffractive lens with overlapping apertures with focal length 500µm, focal pitch 44µm and NA 0.1 for the wavelength 0.5 µm.*

Due to the focal modulation, the Strehl criteria is not suited to assess the quality of the foci. For higher numerical apertures the focal profile remains smooth even when post-processing is applied, therefore the Strehl criteria stays valid.

8 Conclusion

Diffractive lens with overlapping apertures enable a dense spot-wise illumination gird. The realization with continuous amplitude and phase concentrates light as good as the perfect wave. Potential post-processing steps due to fabrication restrictions can be treated as an aberration to an ideal case and result only in a constant multiplicative loss for high numerical apertures. So especially applications like direct laser writing and scanning microscopy which use these high numerical apertures versions benefit from the gained parallelization.

References

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