Diffraction rewind: inversion of signals to initial stress profiles using the 3D optoacoustic Volterra integral

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We consider the mathematically challenging inverse optoacoustic (OA) problem in the paraxial approximation of the underlying wave equation. We illustrate the validity and discuss the limitations of a numerical inversion scheme for the reconstruction of initial stress profiles from OA signals.

1 Introduction

Optoacoustics (OAs) can be considered a two-part phenomenon, based on an initial *optical absorption* of laser pulses by media followed by photothermal heating of the absorber, and, *emission of acoustic stress waves*, triggered by the thermoelastic expansion induced by the heating. During the course of their propagation, the laser-excited stress profiles experience a shape transformation due to diffraction [1]. As far as diffraction is concerned one can distinguish two major problems: (P.1) the *direct* optoacoustic (OA) problem, concerned with the calculation of diffraction-transformed pressure signals at a desired field point for a given initial stress profile, and, (P.2) the *inverse* OA source reconstruction problem, where the aim is to recover initial stress profiles from observed OA signals [2]. The latter is a prime example of an inverse problem, effectively aiming at reconstructing "internal" properties from "external" measurements only.

Owing to its immediate relevance for medical applications, current progress in the field of inverse optoacoustics is driven by photoacoustic tomography (PAT) and imaging applications [2]. While the inversion input for PAT backpropagation approaches consits of a multitude of signals recorded on a surface enclosing the OA source volume, we here describe an alternate approach that relies on "single-shot" measurements, only. Therefore we consider the OA problem in a paraxial approximation to the full wave equation where the diffraction transformation of signals is described by a Volterra integral equation of the 2nd kind [1, 3], for which efficient numerical inversion schemes exist [4, 5].

After illustrating the numerical procedure in the paraxial approximation, we assess how well the inversion protocol carries over to more prevalent optoacoustic problem instances, featuring an exemplary reconstruction for the full OA wave-equation.

2 Optoacoustic signal generation in the paraxial approximation

The propagation of laser-excited acoustic stress profiles $p_0(\vec{r})$ is governed by the scalar OA wave equation

$$\left[\partial_t^2 - c^2\Delta\right] p(\vec{r},t) = \partial_t\, p_0(\vec{r})\,\delta(t), \qquad (1)$$

which yields the excess pressure field $p(\vec{r},t)$ at time t and field point $\vec{r}$ within a medium of homogeneous speed of sound c [1, 2]. In the paraxial approximation, it reduces to the simpler parabolic diffraction equation $[\partial_\tau\partial_z - (c/2)\Delta_\perp]\,p = 0$ [1, 3], and the time-retarded OA signal $p_{\rm D}(\tau)|_{\tau=t+z_{\rm D}/c} \equiv p(\vec{r}_{\rm D},t)$ at the field point $\vec{r}_{\rm D} = (0,0,z_{\rm D})$ along the beam axis with Gaussian transverse profile ($1/e$-width $a_{\rm B}$) can be computed from the initial stress $p_0(\tau)|_{\tau=z/c} \equiv p_0(\vec{r}_\perp = 0, z)$ via [3]

$$p_{\rm D}(\tau) = p_0(\tau) - \int_{-\infty}^{\tau} {\rm K}(\tau-\tau')\,p_0(\tau')\,{\rm d}\tau', \qquad (2)$$

i.e. a Volterra integral equation of 2nd kind where the integral operator features a difference kernel ${\rm K}(x) = \omega_{\rm D}\exp\{-\omega_{\rm D}x\}$, specifying the diffraction transformation of a propagating OA signal with characteristic frequency $\omega_{\rm D} = 2c|z_{\rm D}|/a_{\rm B}^2$.

3 Forward and inverse solution of the OA Volterra equation

We accomplish the forward solution of Eq. (2) by marching in time using our memoization approach detailed in Ref. [5]. For a discretized setting with constant mesh width Δ and $\{t_i\}_{0\leq i\leq N}$ with $t_0 = 0$, $t_i = t_{i-1} + \Delta$, and t_N large enough to ensure a reasonable measurement depth, the forward solver terminates in time $O(N)$. For OA signals $p_{\rm D}$, the source reconstruction problem is solved using a Picard-Lindelöf "correction" scheme, based on the continued refinement of an initial "predictor" $p_{\rm PL}^{(0)}(\tau)$ upon iterating

$$p_{\rm PL}^{(n+1)}(\tau) = p_{\rm D}(\tau) + \int_{-\infty}^{\tau} {\rm K}(\tau-\tau')\,p_{\rm PL}^{(n)}(\tau')\,{\rm d}\tau'. \qquad (3)$$

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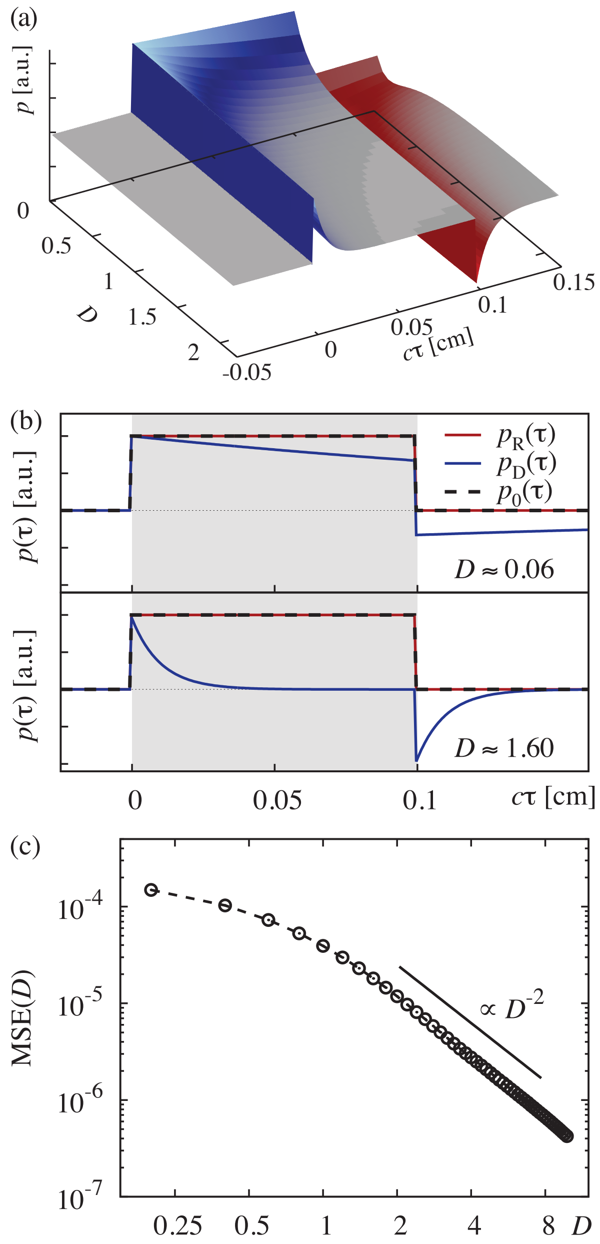

Fig. 1 *Direct and inverse solution of the OA Volterra integral [Eq. (2)] for a box-shaped initial stress profile with* $p_0 = 1$ *in the range* $z = 0 - 0.1$ cm *and beam-width* $a_B = 0.1$ cm. *(a) diffraction transformation of* p_0 *upon propagating from the acoustic near-field (NF) to the far-field (FF). (b) exemplary NF (top) and FF (bottom) inversion via successive approximations. (c) mean-squared-error* MSE *between* p_0 *and* p_R.

The iteration procedure was terminated as soon as max-norm $c_n \equiv \|p_{PL}^{(n+1)}(\tau) - p_{PL}^{(n)}(\tau)\| \leq 10^{-6}$ and the final estimate is referred to as p_R. Albeit more elaborate choices are possible (see Ref. [5]), we here opt for a low-precision predictor by setting $p_{PL}^{(0)} \equiv 0$.

A sequence of numerical experiments, detailing the forward and inverse solution of the OA Volterra integral Eq. (2) for a (dimensionless) box-shaped initial stress profile with $p_0 = 1$ in the range $z = 0 - 0.1$ cm and beam-width $a_B = 0.1$ cm, is illustrated in Fig. 1. Therein, Fig. 1(a) shows the diffraction transformation of p_0 upon propagating from the acoustic near-field (NF) attained in the diffraction parameter range $D \approx 3.2|z_D| < 1$, to the far-field (FF) at $D > 1$, Fig. 1(b) summarizes an exemplary inversion via successive approximations in terms of Eq. (3), and, Fig. 1(c) shows the mean-squared-error MSE between p_0 and inverse estimate p_R, with p_D computed via an independent solver for the full OA wave equation Eq. (1).

Note that a `Python` implementation of our code for the above simulations (detailed in Ref. [5]) can be found at Ref. [6].

4 Summary and Conclusions

We explored the validity and the limits of a numerical inversion scheme for the OA Volterra integral equation by considering synthetic inversion input resulting from box-shaped initial stress profiles. The inversion protocol is exact for the parabolic diffraction equation and asymptotically correct for the full OA wave equation. As evident from our numerical experiments, the discrepancy between the inverse estimate and the true initial stress profile vanishes algebraically upon approaching the far-field limit.

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