Simulative determination of the wavefront uncertainty of PTB's Sphere Interferometer II

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A raytracing-approach for determining the contribution of wavefront deformations to the measurement uncertainty of PTBs new sphere interferometer is presented. The raytracing was used for analysing the uncertainty influence of the alignment procedure and the real sphere topographies to the measurement result.

1 Introduction

The Avogadro-constant N_{A} can be determined via the XRCD method [1]. This method requires the determination of the volume of silicon single crystal spheres which can be calculated from the complete diameter topographies of the spheres [2]. These topographies are measured with the sphere interferometers at the PTB.
The uncertainty budget for the volume determination [2] is dominated by the uncertainty brought into the measurement by unknown wavefront deformations. These are deviations of the wavefront from the optical design which can originate from alignment or manufacturing errors and refractive index inhomogeneities. This uncertainty contribution is not analytically feasible in a reasonable way and therefore only roughly estimated. To overcome this an optical raytracing was implemented that offers a better quantification of the uncertainty.

2 Measurement Principle and Setup

The measurement principle used to measure the diameter topography is shown in Fig. 1.

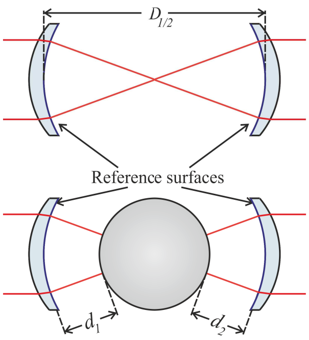

Fig. 1 *Schematic drawing of the centre of the measurement setup*

The distances $D_{1/2}$ and $d_{1/2}$ are measured interferometrically with a phase-shifting technique [3] in spherical etalons of the Fizeau type. These etalons are established by the two opposing reference surfaces for measuring $D_{1/2}$ or one reference surface and a sphere section for the measurement of $d_{1/2}$.

From these measured etalon lengths the diameter can be calculated as

$$d_{\text{sphere}} = \frac{(D_1 + D_2)}{2} - d_1 - d_2. \qquad (1)$$

The spherical design offers the possibility to measure several thousand diameter values in the $45.6°$ field of view (new sphere interferometer). The number of measurement points only depends on the cameras and the optical resolution. To cover the full surface with diameter measurements the sphere can be rotated around two axes. Since this interferometric measurement technique only gives the fractional part of the sphere diameter when expressed in wavelengths it is necessary to determine a preliminary volume of the sphere by measuring its mass and the density for the integer part.

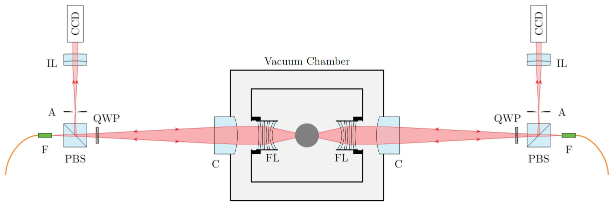

Fig. 2 *Schematic drawing of the complete optical measurement setup*

The complete optical measurement setup is shown in Fig. 2. The light used for the measurements emerges from the multimode fibers F. It is linearly polarized by the polarizing beam splitter (PBS) and subsequently circularly polarized by the quarter wave plate (QWP). The collimator (C) forms the diverging rays to a plane wave which is then focussed to the centre of the etalon by the Fizeau-lenses (FL). The inner surface of the Fizeau lenses also acts as the reference surface for the interferometric measurements. The reflections from the etalon follow back the same way, become again linearly polarized by the QWP so that they are reflected to the CCD inside the PBS. The aperture (A) which lies exactly at the focus acts as a spatial frequency filter. After filtering the light is imaged to the camera by the imaging-lens (IL).

DGaO Proceedings 2016 - http://www.dgao-proceedings.de - ISSN: 1614-8436 - urn:nbn:de:0287-2016-P008-9
submitted: 13.Jun.2016 - published: 5.Aug.2016

3 Raytracing

In the ray-tracing software which is implemented in C++ the path of the rays is calculated from the fibre end surface to the camera surface.
To consider the spatial extent of the fibre end surface in the raytracing multiple source points are used. Also the multiple reflections inside the etalon are traced up to a selectable order of reflection. The wavelength change for the five phase-steps is implemented, too. Additionally the GNU Multiple Precision Arithmetic Library was used which delivers data types with arbitrarily high numerical precision, so that any numerical influence on the raytracing is eliminated. For analysing the uncertainty of the alignment procedure a Monte-Carlo-Method for varying the positions and orientations of several optical elements was implemented which follows the real alignment procedure with its inherent uncertainties. For the determination of the influence of the imperfect spherical shape measured topographies of real spheres parameterized as a set of real spherical harmonics can also be put into the simulation.

4 Analysis of Ray Data

The ray data is analysed in IDL (Interactive Data Language). There a fit of Zernike polynomials to the optical paths of the fields of each source point and each reflection inside the etalon is carried out to obtain the phase in the camera-plane. Afterwards the phase data of all reflections of one source point is coherently overlaid considering the different reflection intensities. This gives the interferograms of each source point. The measured interferogram then is the incoherent superposition of the interferograms of all source points. These steps have to be carried out for all wavelengths and for all four etalon configurations (measurement of $D_{1/2}$ and $d_{1/2}$). The following evaluation of the resulting interferograms is the same as for the real measurements.

5 Results

Fig. 3 *diameter uncertainty due to alignment uncertainties in the measured aperture; Min(blue) =* 0.09 *nm / Max(white) =* 0.23 *nm*

For determining the influence of the alignment procedure measurements with 5000 randomly misaligned systems were simulated. In the systems a perfect sphere was used and only the measurement of one sphere section was calculated. The resulting pixelwise standard deviation equals the expectable uncertainty and is shown in Fig 3. The minimum of the uncertainty lies at 0.09 nm (blue) in the center and its maximum at 0.23 nm (white) at the aperture border. The diameter uncertainty has a mean value of 0.15 nm. This corresponds to a relative volume uncertainty of $5 \cdot 10^{-9}$.
The influence of a real sphere diameter topography was determined by using the measured topography of the natural silicon sphere Si-PTB-11-05 with the deviations from the sphere multiplied by 3.5.

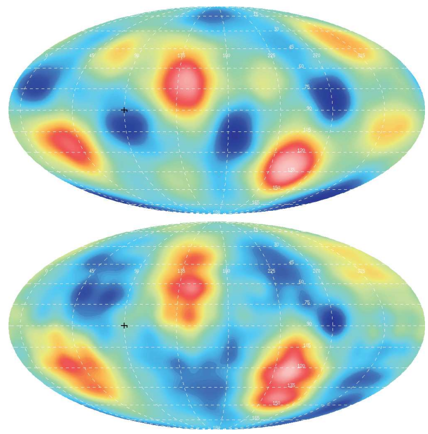

Fig. 4 *top: input topography (PV=150.6 nm), bottom:* $d_{input} - d_{result}$ *(PV=*0.95 *nm, mean=* 0.04 *nm)*

The local deviations between the input and the resulting topography range from -0.37 nm to 0.58 nm with a mean of 0.04 nm and show a slight dependence on the input topography. The determined influence of the imperfect spherical shape on the mean diameter is much smaller than other uncertainty sources like the alignment procedure or the temperature measurement uncertainty [2].

6 Conclusion

The influence of the two adressed types of wavefront deviations is much smaller than expected. This raises the assumption that their uncertainty is overestimated. Including further optical imperfections in the simulation like optical inhomogeneities can provide more evidence.

References

[1] P. Becker, B. Andreas, Y. Azuma, *et al.*, “Counting the atoms in a ^{28}Si crystal for a new kilogram definition,” Metrologia **48**, S1–S13 (2011).

[2] G. Bartl, H. Bettin, M. Krystek, *et al.*, “Volume determination of the Avogadro spheres of highly enriched ^{28}Si with a spherical Fizeau interferometer,” Metrologia **48, No. 2**, S96–S103 (2011).

[3] R. A. Nicolaus, “Precise method of determining systematic errors in phase-shifting interferometry on Fizeau interferences,” Applied Optics **32**(31), 6380–6386 (1993).

DGaO Proceedings 2016 - http://www.dgao-proceedings.de - ISSN: 1614-8436 - urn:nbn:de:0287-2016-P008-9
submitted: 13.Jun.2016 - published: 5.Aug.2016