# Shape reconstruction of 3D objects from noisy slope data


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A specific class of 3D sensors measures the local slope of an object surface. In order to obtain the object's shape, it has to be algorithmically reconstructed from the slope. This is a challenging task since the data is often corrupted by noise or outliers. We present a novel shape reconstruction method which yields both the global shape and the local surface structure of an object with high accuracy.


## 1 Introduction

In this paper we present a novel method to reconstruct the shape of an object from noisy gradient field data. Measuring the slopes of an object instead of its height has several advantages. One of them is the higher information efficiency, since transmitting high frequency data yields a better exploitation of the channel capacity [1]. Examples for slope-measuring sensors are Photometric Stereo, Shearing Interferometry, or Deflectometry [2].

In most applications, however, the shape of an object is of interest. In order to obtain the shape, the gradient field acquired by the sensor has to be integrated.

The existing methods for gradient field integration can be divided into two classes [3]: *Local* methods based on curve integrals accumulate the slopes along a chosen path. However, noise is propagated and thus introduces a shape error along the path. *Global* methods try to minimize a global error functional. This leads to a partial differential equation which is then solved numerically, e. g. by Fourier expansion. These methods imply additional boundary conditions which often can not be satisfied in practice. The Frankot-Chellappa method, for example, requires a periodic extension of the boundaries [4]. If the boundaries are too steep, the reconstruction fails altogether.

## 2 Analytic slope interpolation

We present a shape reconstruction method which is a combined local and global approach. The measured slope values are matched with the slopes of an analytic interpolant. The interpolating function itself then yields the object's shape. The method is based on Hermite-Birkhoff interpolation employing so-called radial basis functions (RBFs). As basis functions, we use Wendland functions of sufficient smoothness [5]. They have limited support; hence, the radius of the basis function controls the locality and the stability of the interpolation (Fig. 1).

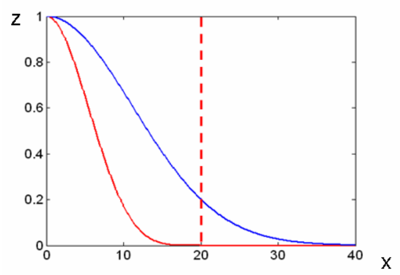


***Fig. 1*** *In contrast to the Gaussian function (blue), the Wendland function (red) has a limited support window.*

The advantage of an interpolation approach is that local details are preserved. Still, the method is stable against noise, since the basis functions yield an energy-minimized surface representation without waviness [6]. Further, it can be applied to grid-free data and thus can handle holes or even ring-shaped topologies.

## 3 Reconstruction of large data sets

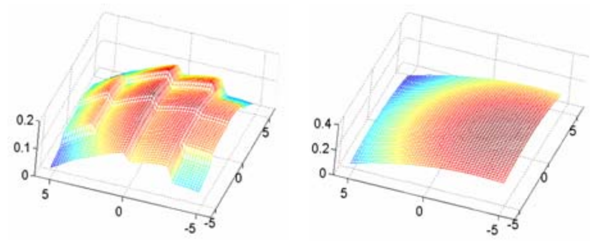

***Fig. 2*** *Left: The height is reconstructed on overlapping patches up to an integration constant. Right: A least-squares fit has been applied to obtain the object's overall shape (all units in mm).*

Performing the interpolation requires solving a linear system of equations. A typical data set consists of up to one million gradient vectors and thus leads to a system matrix of size two-by-two million.

To cope with such huge data sets, we developed a two-step method. First, we split the gradient field in overlapping patches and interpolate the data on each patch separately. After this step, the object's height is known up to a constant of integration. In the second step we determine this height constant using a least-squares fit on the overlaps (Fig. 2).

## 4 Simulation results

We simulated the measurement of a sphere of radius 80 mm on a field of 80×80 mm². We added uniformly distributed noise of 10 arcsec, which is realistic sensor noise. The lateral resolution was 0.2 mm. Fig. 3 displays the difference between the ideal sphere and the reconstructed one. The global deviation from the ideal sphere is less than 100 nm, the local error is below 10 nm.

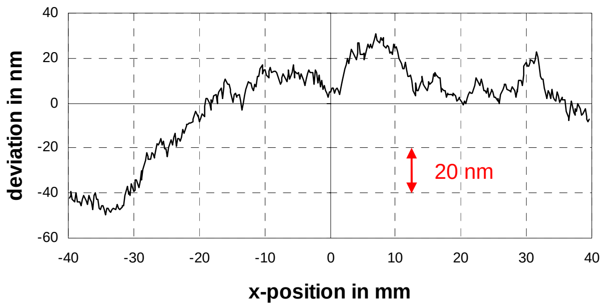


***Fig. 3*** *Cross-section of the deviation of a reconstructed sphere from the ideal sphere in a numerical simulation.*

## 5 Experimental results

We measured an object slide with a groove of about 0.5 µm with Phase-Measuring Deflectometry (PMD) and reconstructed the shape applying our new method.

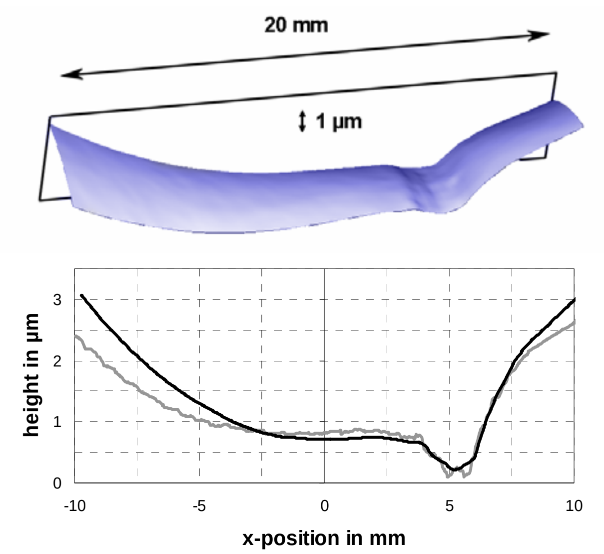


***Fig. 4*** *Comparison of the reconstructed height measured with PMD (black) and a white-light interferometry measurement (gray) of an object slide with a groove.*

Fig. 4 depicts the comparison of our reconstruction with a height profile obtained by white-light interferometry. Their difference is below 1 µm on a field of 20×20 mm².

Fig. 5 shows the fine-structure on a plastic micro-lens. The global shape has been subtracted to make fine details visible, thus the image appears "rendered". The depicted part of the lens has a width of 0.5 mm, the clearly visible groove is about 30 nm deep. This demonstrates that small details are preserved by the reconstruction algorithm.

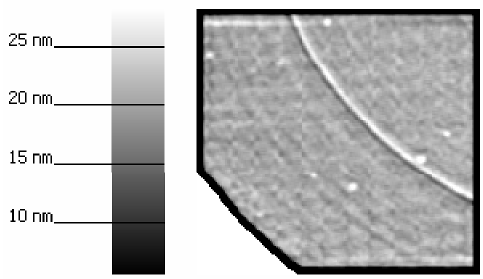


***Fig. 5*** *Fine structure of a plastic micro-lens.*

## 6 Conclusion

We have shown that our new method is able to reconstruct surfaces from noisy gradient fields both globally and locally with high accuracy. It can handle incomplete and grid-free data sets and it can be applied to a large class of objects.

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## References


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