Miscellaneous about the simulation and evaluation of foci

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The intensity distribution in the focus of a lens is influenced by many parameters like e.g. the wavelength and the numerical aperture of a lens. But, also other parameters are important like the phase and amplitude of the pupil function, polarization effects or diffraction at the exit pupil (for small Fresnel numbers).

1 Introduction

In 1835 Airy derived the formula for the intensity distribution in the focus of an ideal diffraction-limited lens with homogeneous illumination and circular aperture (using the approximation of scalar wave optics). He calculated that the radius from the center to the first minimum is

$$r = 0.61 \frac{\lambda}{\mathrm{NA}} \qquad (1)$$

where λ is the wavelength of the light and NA the numerical aperture of the lens. A similar expression is used in modern lithography to describe the resolution of a lens by replacing the constant value 0.61 by a factor *k* which indeed can summarize all effects of the illumination and aberrations of the lens.

2 Influence of aberrations

It is well-known that the phase deviations of the pupil function, i.e. the wave aberrations *W*, have a strong influence on the intensity distribution in the focus of a lens. By using ray tracing to calculate the wave aberrations there can be defined at least three different types of foci: the paraxial focus, the best focus with respect to minimum lateral ray aberrations and the best focus with respect to minimum wave aberrations (i.e. optical path differences). Here, minimum ray or wave aberrations mean that the root mean square value (rms) of the aberrations has a minimum. In general, the different foci are all situated in different planes. The intensity distribution *I* can be easily calculated by the well-known point spread function formula assuming a lens with large Fresnel number and small NA (neglecting polarization effects):

$$I \propto \left| \mathrm{FFT}\left\{ A e^{ikW} \right\} \right|^2 \qquad (2)$$

$k=2\pi/\lambda$ is the wave number, FFT symbolizes a Fourier transformation (and in the numerical case a fast Fourier transformation) and the wave aberrations *W* have to be calculated by taking into account the defocus terms of the different foci, respectively. Here, the amplitude function *A* is constant (homogeneous illumination). For the example of primary spherical aberration Fig. 1 shows a ray tracing simulation in the focal region of a lens and Fig. 2 shows the intensity distributions in the different foci. For small aberrations the best focus concerning wave aberrations has also the largest Strehl ratio, but for this example with a quite high Zernike coefficient of 0.4 wavelengths spherical aberration both "best foci" have about the same Strehl ratio.

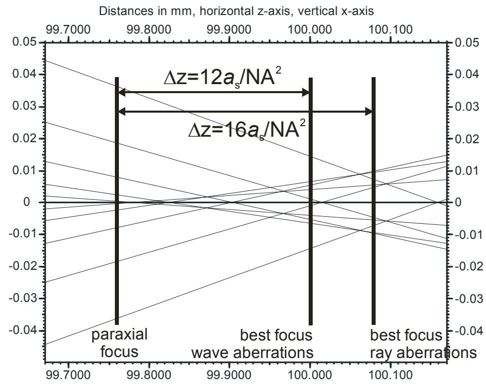

Fig. 1 *Rays and different foci in the focal region of a lens having primary spherical aberration with a Zernike coefficient $a_s=0.4\,\lambda$,* NA=*0.1 and focal length f=100 mm.*

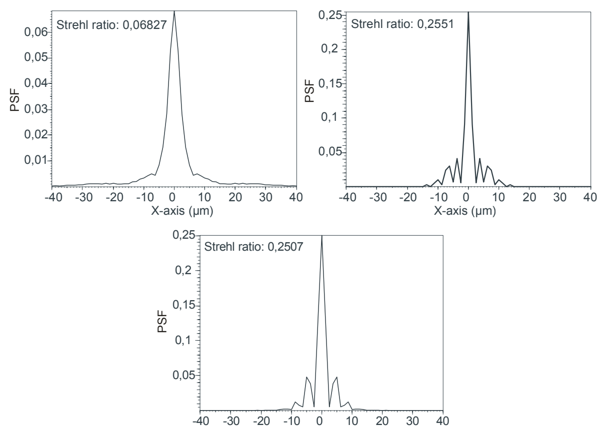

Fig. 2 *Intensity distributions for spherical aberration: paraxial focus (top left), best focus concerning ray aberrations (top right), best focus concerning wave aberrations (bottom).*

3 Influence of the amplitude function

The focus is of course not only influenced by the wave aberrations *W* in the exit pupil but also by the amplitude function *A*. This can easily be seen by the example of a single lens with high NA having a plane and an aspheric surface. The aspheric surface can either be the front or the back surface and in both cases the aspheric surface is designed in such a way that the wave aberrations of the lens are zero for an incident on-axis plane wave (see Fig. 3). But it can also be seen that in the case of the aspheric front surface (left) the amplitude function will decrease towards the rim (the rays are less close to each other behind the lens), whereas in the case of the aspheric back surface (right) the amplitude increases towards the rim. In both cases it is assumed that the surfaces are anti-reflection coated and only a cylindrical lens (1D simulation) is regarded. Instead of using equation (2) the simulation can also be done by the scalar wave propagation method (WPM) [1]. The intensity distribution in the focal plane simulated with the WPM can be seen in Fig. 4. For the aspheric back surface the increase of the amplitude towards the rim acts similar to an annular aperture and causes a smaller central maximum but higher side lobes.

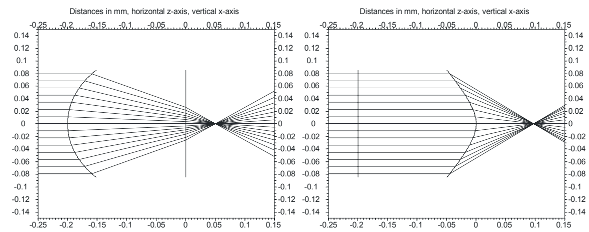

Fig. 3 *Ray tracing schemes of the two aspheric lenses without aberrations but with different amplitude function.*

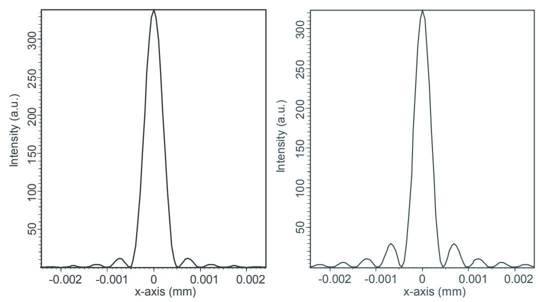

Fig. 4 *Intensity in the focal plane (1D simulation for a cylindrical lens) for the aspheric front surface (left) and aspheric back surface (right). In both cases, the numerical aperture is 0.5 and the lens has no aberrations.*

4 Polarization effects

For lenses with quite high numerical apertures of more than about 0.5 or for special polarization patterns of the incident light (e.g. radially polarized light) it is no more valid to use a scalar wave-optical simulation because the electric and magnetic field have to be added like vectors. To calculate the electric energy density in the focus the vectorial formulation of the Debye integral can be used [2]. Here, a numerical formulation is taken which superimposes all plane waves traveling along the path of a light ray from the exit pupil to the focus. Since the shape of the exit pupil has to be known it is only implemented for a lens fulfilling the sine condition (exit pupil is a sphere around the focus) or for an idealized flat lens (like an idealized diffractive optical lens with 100% efficiency) [3].

5 Lenses with small Fresnel numbers

For lenses with very small Fresnel numbers *F* (*F*<10) it has to be taken into account that the maximum intensity on-axis is not in the focal plane of the lens but closer to the lens, because the diffraction at the exit pupil acts like an "additional Fresnel zone" (but this effect is not linear and for *F*>10 it nearly vanishes).

6 Evaluation criteria for the spot size

For the evaluation of the quality of a certain spot the special properties of the detector have to be considered. It is e.g. a big difference whether the radius with 84% encircled energy or the radius from the center to the first minimum is taken, although both values are identical for an ideal scalar Airy pattern (see Fig. 5, where the radius with 84% encircled energy of the flat lens reaches a minimum for a numerical aperture of about 0.5).

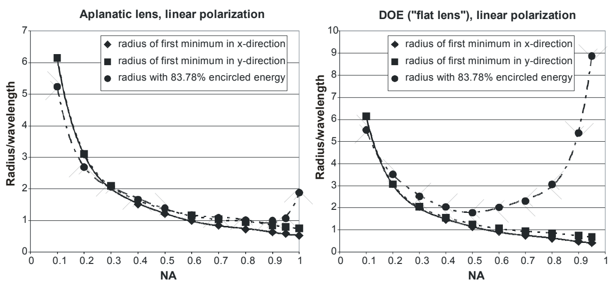

Fig. 5 *Focus radius from the center to the first minimum (in x- and y-direction) or with 84% encircled energy for an ideal aplanatic lens (left) or an ideal flat lens (right) for an incident plane wave polarized in y-direction.*

7 Acknowledgement

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References

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[2] B. Richards, E. Wolf, „Electromagnetic diffraction in optical systems" in *Proc. R. Soc. A* **253**: 358-379 (1959)

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