Polarisation properties of perfect high-NA focusing lenses

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We present a new approach for calculating the exact electric field distribution behind a perfect high NA focussing lens with an aspheric front and spheric back surface. The obtained results are used to find the optimal shape of the lens with respect to light efficiency and polarisation dependence.

1 Introduction

High NA lenses are very important in the field of microscopy and optical storage. For numerical apertures larger than 0.5, the polarisation properties are relevant. Common vector approaches to focusing assume a spherical wave front, on which the field amplitude is constant. We have derived a perfect lens model, which allows to also treat reflection and transmission at the lens surface.

In a previous work [1], we have developed an analytic description for a perfect plano-convex focussing lens. We have extended this theory to perfect lenses with a spherical back surface and we use a raytracing approach to calculate the vectorial fields at the surfaces of a high-NA lens. After that, we use the Debye integral to determine the focal distribution. The obtained results can be used to optimise the lens shape with respect to light efficiency and polarisation dependence.

2 Theoretical basis of optimisation

The geometry of the system is represented in Fig 1. For a perfect focusing lens, all the rays parallel to the optical axis have to be focused into one geometric spot. According to the geometry of the system the minimum positive radius of the back surface is given:

$$RBS_{\min} = \frac{w \cdot NA}{1 - NA} \qquad (1)$$

By applying the vectorial law of refraction, the weighted direction vector inside the lens ($\vec{t}_1$), as well as the normal of the back-surface ($\vec{N}_B$) and the normal to the front-surface ($\vec{N}_F$) can be computed from the vector $\vec{t}_2$:

$$\vec{t}_1 = \vec{t}_2 + \vec{N}_B \left(\sqrt{n_1^2 - n_2^2 + \left(\vec{t}_2 \cdot \vec{N}_B\right)^2} - \vec{t}_2 \cdot \vec{N}_B \right) \qquad (2)$$

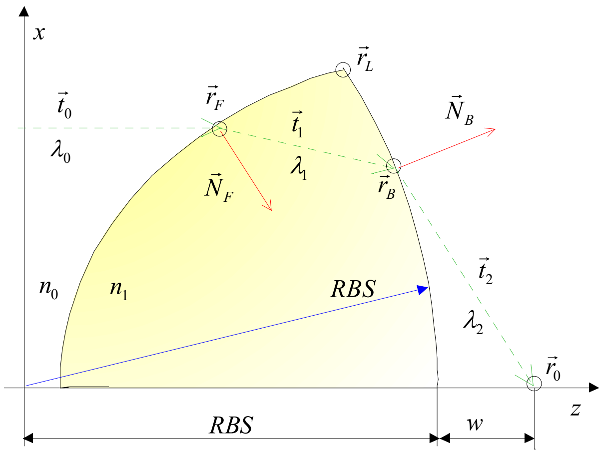

Fig 1. *Geometry of the optical system*

$$\vec{N}_B = \frac{\vec{r}_B}{\left|\vec{r}_B\right|} = \frac{\vec{r}_0 - \lambda_2 \vec{t}_2}{\left|\vec{r}_0 - \lambda_2 \vec{t}_2\right|} \quad , \quad \vec{N}_F = \frac{\vec{t}_1 - \vec{t}_0}{\left|\vec{t}_1 - \vec{t}_0\right|} \qquad (3)$$

For the parameter λ_2 we found an analytic solution:

$$\lambda_2 = \vec{r}_0 \vec{t}_2 - \sqrt{\left(\vec{r}_0 \vec{t}_2\right)^2 - \left(\vec{r}_0^{\,2} - RBS^2\right)} \qquad (4)$$

With these parameters, the electric field distribution inside the lens can be determined analytically from the input electric field ($\vec{E}_0$), by decomposing the field into s- and p-components and using the Fresnel coefficients for transmission:

$$\vec{E}_1 = \left(\left(\vec{E}_0 \cdot \vec{e}_{s0}\right) \cdot t_{s0}\right) \cdot \vec{e}_{s0} + \left(\left(\vec{E}_0 \cdot \vec{e}_{p0}\right) \cdot t_{p0}\right) \cdot \vec{e}_{p1} \qquad (5)$$

where the unit vectors $\vec{e}_{s0}, \vec{e}_{p0}$ and $\vec{e}_{p1}$ are computed from the surface normal and the incident vector. The electric field behind the focusing lens can be also determined analytically:

$$\vec{E}_2 = \left(\left(\vec{E}_1 \cdot \vec{e}_{s1}\right) \cdot t_{s1}\right) \cdot \vec{e}_{s1} + \left(\left(\vec{E}_1 \cdot \vec{e}\,'_{p1}\right) \cdot t_{p1}\right) \cdot \vec{e}_{p2} \qquad (6)$$

where the unit vectors $\vec{e}_{s1}, \vec{e}_{p1}$ and $\vec{e}_{p2}$ are also computed from the surface normal and the incident vector. The correction term, included in [2], here is implicitly included in the transmission coefficients.

Fig 2. shows the x-, y- and z-component of the electric field distribution behind the focusing lens. We assumed a linear x-polarised field at the input. The top row shows the distribution obtained by using the common approximation [2]. The bottom row shows the exact result, obtained with our method assuming a planar back surface. The results become identical, if the transmission coefficients are set to unity.

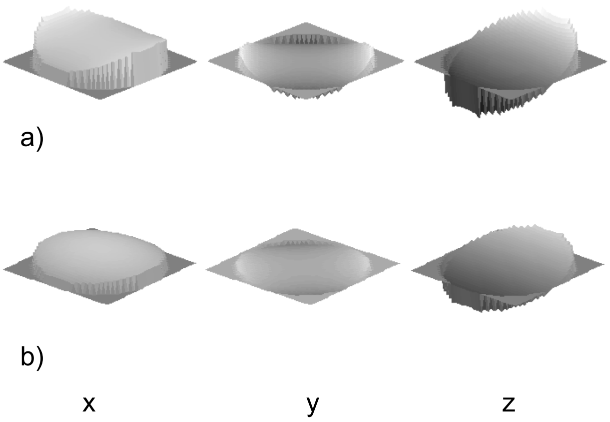

Fig 2. *Comparison of the electric field distribution components behind the focussing lens: (a) ref [2], (b) this theory*

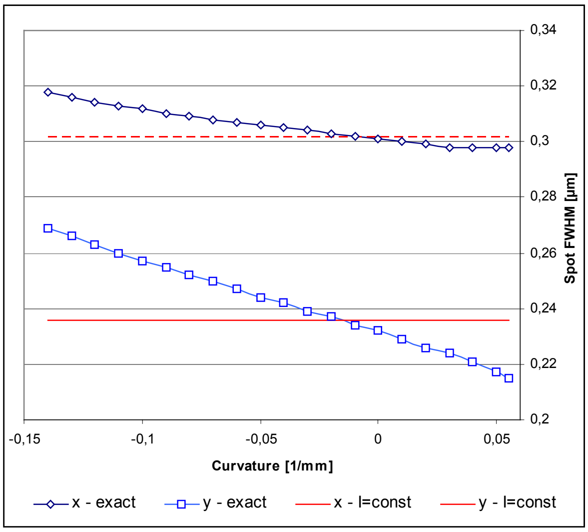

Fig 3. *Spot FWHM in x- and y-direction as a function of the back surface curvature*

Fig. 3 shows a comparison of the spot FWHM as a function of the back surface curvature. The calculation was done for linear x-polarisation input. It can be seen, that the spot widths depends on the back surface curvature and thus can be optimised.

For comparison, the spot size computed with constant pupil illumination [2] is also shown in the figure. Interestingly, the ellipticity of the spot also changes with varying back surface curvature.

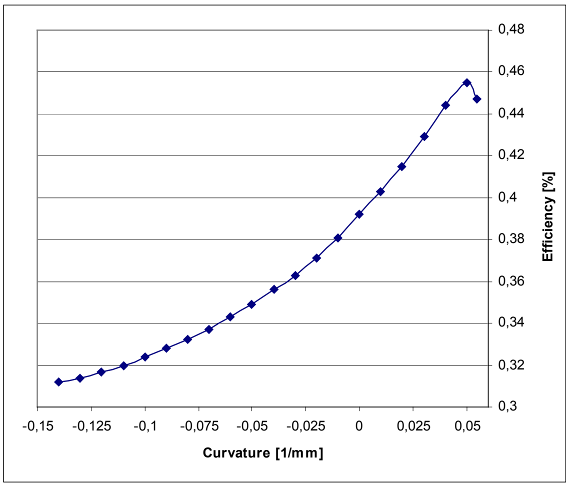

Fig 4. *Total light efficiency as a function of the back surface curvature*

Fig 4. shows the total light efficiency as a function of the curvature of the back surface. The NA and the focal length in this comparison are held constant. It can be seen, that there is an optimum for a back surface curvature of $0,05 mm^{-1}$, assuming $NA = 0,85$ and $w = 3mm$.

3 Conclusion

We have presented a new analytic approach for modelling perfect high NA focusing lenses. The approach provides – but does not require knowledge about the shape of the aspherical front surface. Only the curvature of the spherical back surface is needed. We have compared our results to those obtained from existing theories. Using this approach, we have analysed the spot FWHM and the total light efficiency as a function of the back surface curvature. We have shown, that both are dependent on the back surface curvature. Results can be applied to lens optimisation in microscopy or optical storage.

References

[1] K.–H. Brenner, "Polarisation analysis of an ideal plano-convex focusing lens", Annual Report, Chair of Optoelectronics, University of Mannheim (2001)

[2] M. Mansuripur, J. Opt. Soc. Am. A/Vol. 10, No. 2/February 1993