Is a right-handed negative refracting material possible?

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Conditions for the dispersion relation and the dielectric function of a right-handed negative refracting material are derived. The refraction at an interface between two materials is studied. It is determined by the boundary conditions for the electromagnetic field at the interface whether negative refraction is possible.

1 Introduction and motivation

Some time ago V.G. Veselago studied theoretically so called left-handed materials [1]. Those are materials with negative dielectricity ε and negative magnetic permeability μ. And he showed that these fictitious materials refract negatively. Left-handedness is a sufficient condition for negative refraction. Therefore, the notions left-handedness and negative refraction are closely related.

However, there exist no left-handed materials in nature. Artificially produced left-handed materials for microwaves, which have been tested up to now only in a few experiments [2], suffer from high absorption losses. An important task is to develop negative refracting materials also for optical frequencies. Therefore, we investigate whether a right-handed (ε>0, μ>0) negative refracting material is also possible. The negative refraction should be in such a way that Snell's law is fulfilled. The absorption of the material should be negligible. How must the dispersion relation $\omega = \omega(\vec{k})$ of the material be in order to achieve these desired properties?

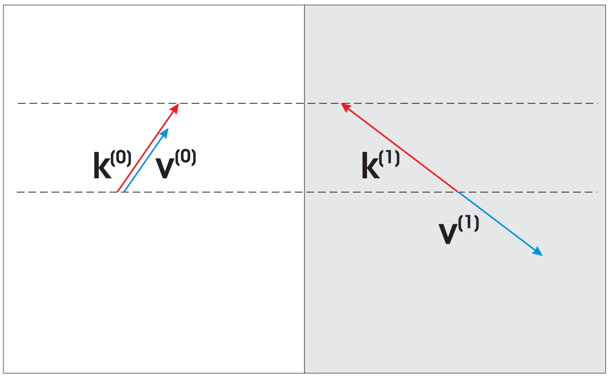

Fig. 1 *Negative refraction at an interface. Incident (refracted) wave with wave vector $\vec{k}^{(0)}$ ($\vec{k}^{(1)}$) and group velocity $\vec{v}^{(0)}$ ($\vec{v}^{(1)}$). The direction of a light ray is given by the group velocity vector.*

2 Properties of the dispersion relation

The negative refraction at an interface is depicted in figure 1. In the negative refracting material the group velocity is antiparallel to the wave vector, that is

$$\frac{\partial\omega(k)}{\partial k} < 0. \tag{1}$$

We assume that the frequency ω depends only on the magnitude of the wave vector $\vec{k}$ but not on its direction. That means the material is isotropic and Snell's law is fulfilled.

3 Properties of the dielectric function

The behaviour of the dispersion relation is mainly governed by the behaviour of the dielectric function. For isotropic, transparent and non-magnetic materials the relation

$$\vec{k}^2 = \left(\frac{\omega}{c}\right)^2 \varepsilon(\omega) \tag{2}$$

holds [3], where the dielectric function ε is real and positive (Im ε=0, Re ε≥0). If ε were not real and positive, the material would not be transparent. For such materials the condition for negative group velocity (1) is equivalent to the inequality

$$\frac{\omega}{\varepsilon}\frac{\partial\varepsilon}{\partial\omega} < -2. \tag{3}$$

The conditions on the dielectric function ε are illustrated in figure 3. We need a frequency interval (marked by the dashed vertical lines in figure 3) on which Im ε is approximately zero. Thus the interval must be far away from the resonances. Re ε must be a decreasing function on this frequency interval in order to fulfil inequality (3). However, real and imaginary part of the dielectric function are not independent of each other. They are related by the Kramers-Kronig relation [3]

$$\mathrm{Re}\,\varepsilon(\omega) = 1 + \frac{2}{\pi}\mathrm{P}\int_0^\infty \mathrm{d}\omega' \frac{\omega'\mathrm{Im}\,\varepsilon(\omega')}{\omega'^2 - \omega^2}. \tag{4}$$

Due to this relation Re ε is an increasing function on frequency intervals on which a passive material (Im ε≥0 $\forall\omega$) is transparent. Therefore, inequality

(3) cannot be fulfilled in a passive material and the behaviour shown in figure 3 is impossible. The inequality can only be fulfilled in a material which is capable of stimulated emission (Im ε≤0).

4 Boundary conditions

For a given incident wave with frequency $\omega^{(0)}$ and wave vector $\vec{k}^{(0)}$ there are two possible refracted waves which have the same frequency ($\omega^{(0)} = \omega^{(1)} = \omega^{(2)}$) and the same wave vector component parallel to the interface ($k_{\|}^{(0)} = k_{\|}^{(1)} = k_{\|}^{(2)}$) (see figure 1 and 2). The refraction is negative in figure 1, whereas it is positive in figure 2. It has been shown theoretically [4] and experimentally [5] that the case depicted in figure 2 can occur though the group velocity vector of the refracted wave points to the interface. It is determined by the boundary conditions for the electromagnetic field – $E_{\|}$ and $H_{\|}$ must be continuous at the interface – which of the two cases is realised. A detailed calculation, which also takes the reflected wave into account, shows that the case depicted in figure 1 occurs for left-handed materials and the case in figure 2 occurs for right-handed materials. Because of the boundary conditions a right-handed negative refracting material is not possible. This result is based on the assumptions that

- the material is homogeneous and isotropic
- there are no surface currents at the interface

Note that because of the assumption of homogeneity photonic crystals are excluded from our considerations. Photonic crystals are regarded as possible candidates for a (right-handed) negative refracting material (see for example [6]).

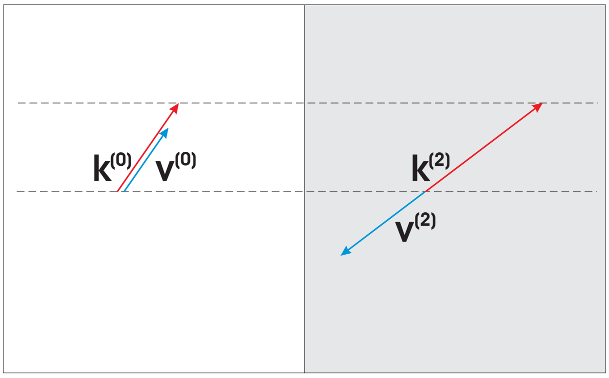

***Fig. 2** Positive refraction at an interface. Incident (refracted) wave with wave vector $\vec{k}^{(0)}$ ($\vec{k}^{(2)}$) and group velocity $\vec{v}^{(0)}$ ($\vec{v}^{(2)}$).*

5 Summary

We have discussed conditions on the dispersion relation and on the dielectric function of a material. However, these conditions are only sufficient for negative group velocity but not for negative refraction. For a proper treatment of refraction the boundary conditions for the electric and the magnetic field must be taken into account. Because of these boundary conditions a right-handed, homogeneous and isotropic negative refracting material is impossible. To achieve negative refraction for right-handed materials it is necessary to give up at least one of the assumptions that were used for the derivation of this result.

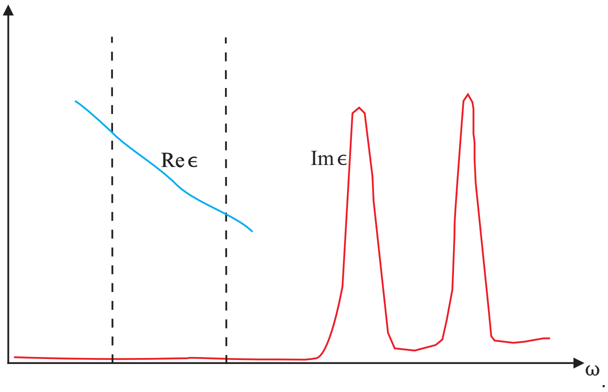

***Fig. 3** Real and imaginary part of the dielectric function. As a consequence of the Kramers-Kronig relation the behaviour of ε shown here is impossible.*

Literatur

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