Numerical modelling of imaging of the eye pupil through the cornea

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In pupilometry one measures and analyses the size of the pupil image observed through the optical system of the cornea. By using ray tracing method we have calculated the influence of the corneal shape, the size and location of the real pupil and the aperture and position of the optical system used to observation on the magnification, position and quality of the pupil image.

1 Introduction

In the experimental pupilometry one measure and analyses the size and shape of the pupil, which is important in clinical diagnosis. Most often it is not taken into account that the pupil is not observed directly, but through the optical system of the cornea. The cornea is the main refractive component of the eye and the influence of its shape should not be neglected in pupilometric measurements [1].

The pupil's shape described in literature often has a complex form. In many cases an ellipse is the best approximation of this shape [2],[3].

According to our calculation the pupil image is magnified, blurred and lays nearer the corneal apex than the real pupil.

2 Method

We have applied the out of axis ray tracing method using MATLAB environment to calculate the properties of the pupil image.

A corneal topography can be approximated by the use of several mathematical functions [4],[5]. The most popular representation of the corneal profile is elongated ellipsoid [5]:

(1) $$z(x,y)=\frac{R_{0x}}{1-\varepsilon_x^2}\left(1-\sqrt{1-\frac{1-\varepsilon_x^2}{R_{0x}}\cdot\left(\frac{x^2}{R_{0x}}+\frac{y^2}{R_{0y}}\right)}\right),$$

where:
R_{0x} – central radius of curvature in *xz* plane,
R_{0y} – central radius of curvature in *yz* plane,
ε_x – the eccentricity in the *xz* plane.

As it is shown in figure 1. the coordinate system is placed in the corneal apex. The large axis of the ellipsoid coincides with the axis of the cornea and lays along the *z* axis of the coordinate system.

The rays beam comes from the point $P(x_p, y_p, z_p)$ on the edge of iris and refract on the cornea. Value of *x* determines the radius of the real pupil, while value of *z* – distance of the pupil from the corneal apex. The angle γ between two extreme rays of the beam is limited by the aperture of a lens *L* of the recording system. There are only these rays considered which fit the objective of the recording system after refraction on the cornea. The aperture radius of this system is equal to x_{ot} and it is distant z_{ot} from the cornea. The virtual pupil image is placed on an extension of refracted rays.

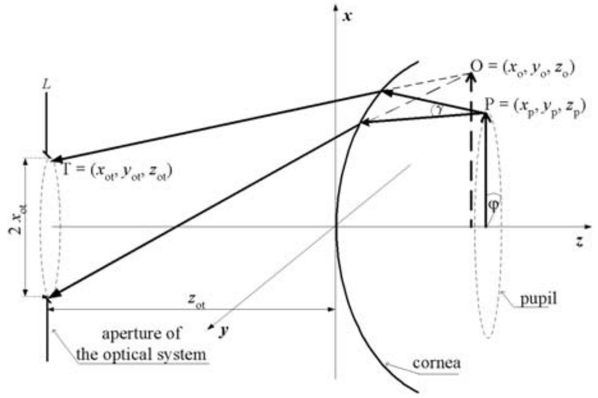

Fig. 1 *The scheme of the image formation of the pupil through the cornea.*

3 Results and discus

Calculations have shown that the position of the centre of gravity becomes stable for about 6000 — 7000 rays for each point of the pupil. It causes that calculations last about 3 hours on a PC computer.

Considering the problem in 2D case, it turned out that each pair of refracted rays on the cornea crosses in different places. The space, where one gets the image of the pupil in 2D looks as shown in the figure 2.

As a blur of the pupil image we mean the size of the spot diagram, where the centre of gravity is an arithmetical average of all *x* and *z* values. The distance between two extreme points (*b* - *a*) changes with parameters of the recording system, the cornea and the pupil.

The greatest influence on the blur of the pupil image has the aperture and the distance of the optical system used to observation from the corneal apex. All further calculations were made for the parameters as follows: x_{ot} = 5mm,

z_{ot} = 150mm. For the greater apertures and smaller distance the blur increases strongly.

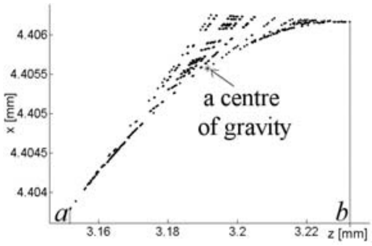

***Fig. 2** Spot diagram representing image of the single point of the pupil.*

The size of the spot diagram, i.e. sharpness of the observed pupil image, also depends on the corneal geometry (Fig. 3). The dependency on the central corneal radius is stronger than on its eccentricity and the sharpness of the observed pupil image decreases for bigger central radius of the cornea. The value of the difference *b* - *a* can reach 0.1mm for the greatest real pupil (x_p = 4mm, z_p = 4mm). As it is shown in the figure 4 the blur of the pupil image decreases with the radius and the distance from the corneal apex of the real pupil.

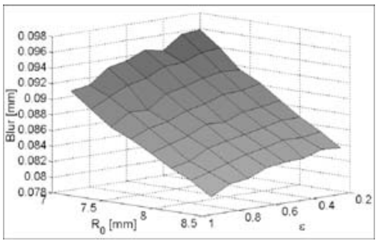

***Fig. 3** Spot diagram size as a function of the corneal geometry for a pupil:* x_p = 4mm, z_p = 4mm.

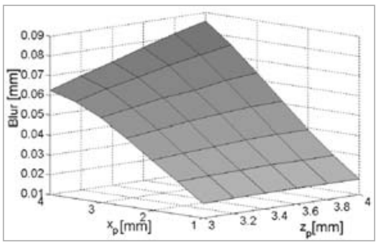

***Fig. 4** Spot diagram size as a function of the pupil diameter and its distance from the corneal apex* (R_0 = 7.8mm, ε = 0.7).

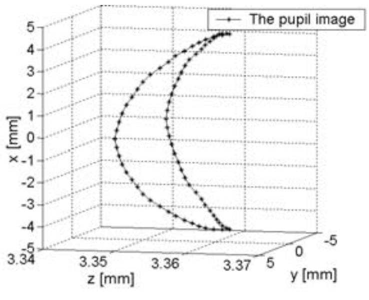

***Fig. 5** A „space" image of the plane pupil formed by non-rotational (astigmatic) cornea.*

Results have also shown that the linear magnification of the virtual image of the pupil can reach 15%, and the difference between the position of the real pupil and the virtual pupil can be 0.7mm. For average parameters of the cornea and pupil (R_0 = 7.8mm, ε = 0.7, x_p = 4mm) [1].

The calculations mentioned above were carried out only for one point of the real pupil. The "space" image of the plain pupil formed by astigmatic cornea with parameters as follows: R_{0x} = 7.8 mm, R_{0y} = 7.5 mm, ε = 0.7 is not a plain any more (Fig. 5). It has the shape of a croissant, what additionally limits the sharpness of the observed pupil image. The projection of that image on the *xz* plain gives an ellipse.

4 Conclusion

As our calculation has shown the image of the pupil is always blured and sharpless. It is also magnified and located closer to the corneal apex than the pupil itself. The greatest influence on blur of the pupil image has the aperture and the location of the observating optical system. Moreover the pupil image observed through the cornea without rotational symmetry forms a curve in space.

References

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