Coupled-Wave Theory for Phase-Shift Keyed Bragg Gratings

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We present the generalization of Kogelnik's theory of coupled waves for the case of the grating comprising several cells along the direction of light propagation. The cells have the same period and amplitude, but different phases. By choosing a set of phases of the cells, a grating with a desired wavelength transfer function can be synthesized. We demonstrate gratings suitable for WDM applications.

1 Introduction

Kogelnik's theory of coupled waves [1] is commonly used to describe the diffraction by simple volume gratings. In a generalization considering spatial phase modulation of Bragg gratings for the first time Kogelnik's theory of coupled waves is applied to phase-shift keyed Bragg gratings, which have been realized experimentally [2] as a flexibly reconfigurable filter with applications in dense wavelength division multiplexing (DWDM) in optical telecommunication. An analytical solution has been derived, which is used to calculate the diffraction efficiency as a function of the wave number for phase-shift keyed Bragg-gratings. A Bragg grating representing a filter to switch on and off selected DWDM channels is presented.

2 Generalization of Kogelnik's theory of coupled waves for phase-shift keying of Bragg gratings

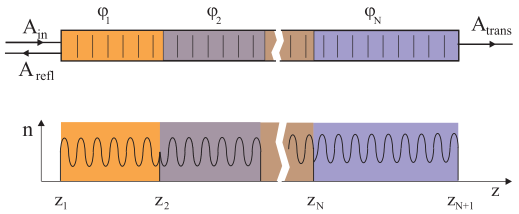

Fig. 1 *Read out geometry and principle of phase-shift keying of Bragg gratings. A_{in}, A_{refl} and A_{trans} are the complex amplitudes of the incident, reflected and transmitted readout beam, respectively. z_m and z_{m+1} for m=1, ..., N are the boundaries of the m-th grating cell having a phase shift of φ_m. n is the refractive index and z is the space coordinate along the grating. N is the number of cells.*

The principle of phase-shift keying and the readout geometry of the Bragg gratings is illustrated in Fig. 1. A phase-shift keyed Bragg grating consists of several cells, which have same grating amplitude and same grating period, but different phases. This results in a refractive index varying along the grating as follows:

$$n(z) = n_0 + n_1 \cos(Kz + \varphi(z)), \qquad (1)$$

where n(z) is the refractive index of the material along the z-axis, K is the spatial frequency of the grating, n_0 is the mean refractive index and $n_1 \ll n_0$ is the grating amplitude. φ(z) is the phase-shift of the Bragg grating at point z:

$$\varphi(z) = \begin{cases} \varphi_1 & \text{for } z_1 < z < z_2 \\ \varphi_2 & \text{for } z_2 < z < z_3 \\ \dots & \\ \varphi_m & \text{for } z_m < z < z_{m+1} \\ \dots & \\ \varphi_N & \text{for } z_N < z < z_{N+1} \end{cases} \qquad (2)$$

where z_m and z_{m+1} are the boundaries and φ_m is the phase-shift of the m-th grating cell. $T=z_{N+1}-z_1$ is the grating length. For the illumination by a plane wave incident perpendicular to the grating planes (see Fig. 1) with wave number ρ inside the medium, the diffraction by the Bragg-grating is described by the following coupled differential equations:

$$R'(z) = -i\kappa \exp(-i\varphi(z)) S(z) \qquad (3)$$

$$S'(z) - 2i\delta \cdot S(z) = i\kappa \exp(i\varphi(z)) R(z) \qquad (4)$$

where R and S are the complex amplitudes of the forward and backward propagating wave along the grating, respectively, R' and S' denote first derivatives, $\kappa=\rho n_1/(2n_0)$ is the coupling parameter, δ is the deviation from the Bragg wave number inside the medium. In comparison to [1] these equations contain the phase factors exp(±iφ(z)) taking account of the phase-shift keying. For each cell the differential equations can be solved separately and by the use of the boundary conditions at the boarders of neighbored cells an analytical solution can be obtained for the diffraction efficiency

$$\eta(\Delta k) = \left| \frac{A_{refl}(\Delta k)}{A_{in}} \right|^2, \qquad (5)$$

where Δk is the wave number deviation from the Bragg wave number in the medium, A_{refl} and A_{in} are the complex amplitudes of the reflected and the incident plane wave, respectively.

3 Bragg filter for switching selected WDM channels

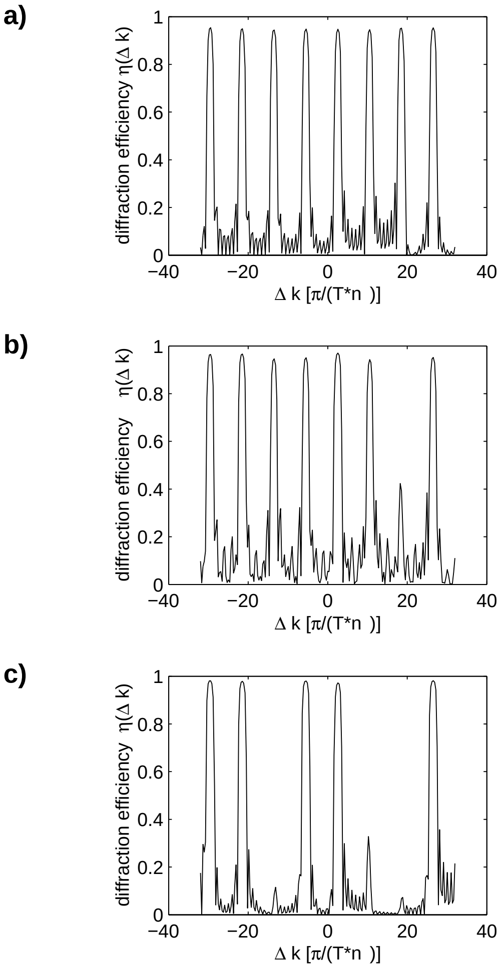

Fig. 2 *Theoretically calculated dependence of the diffraction efficiency η on the wave number deviation Δk from the Bragg wave number for a grating consisting of 64 cells of same length. Parameters: grating length T= 8 mm; mean refractive index n_0=2.3; grating amplitude n_1=4.5·10^{-4}; different phase combinations calculated numerically. For a Bragg wavelength of 1560 nm one unit of the Δk axis is equivalent to a difference of wavelength of 0.066nm.*

By an appropriate phase-shift keying of Bragg gratings using 64 cells of same length a filter can be synthesized, which selects 8 wavelength ranges on an equidistant grid as shown in Fig. 2a. This figure shows the theoretical diffraction efficiency of the filter as a function of the wave number deviation Δk from the Bragg wave number. For a mean refractive index of n_0=2.3 and a grating length of T= 8 mm, calculated in wavelength the maxima have a FWHM of 0.12 nm and spacing of 0.53 nm. By the change of the grating length the spacing can be adjusted to a 0.4 nm grid of DWDM channels defined by the international telecommunication union. The phases of the phase-shift keying for this filter was calculated numerically and is show in Fig. 4. By changing only the phases the filter can be reconfigured to select another set of wavelength ranges as shown in Fig. 2b and c. The reconfiguration between Fig. 2a, b and c can be regarded as switching on and off desired wavelength ranges. Fig. 3 shows that by changing only the phases of the phase-shift keyings also the width of the maxima can be adjusted; in this case to 0.31 nm in wavelength.

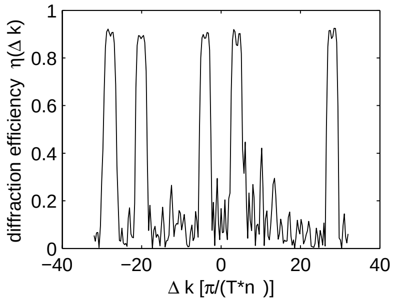

Fig. 3 *Theoretically calculated dependence of the diffraction efficiency η on the wave number deviation Δk from the Bragg wave number for a grating consisting of 64 cells of same length. Same parameters as for Fig. 2 except for the phase combination.*

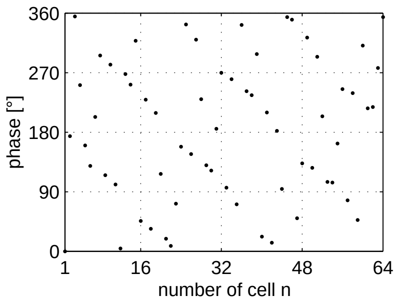

Fig. 4 *Phases of the phase-shift keying corresponding to Fig. 2a.*

4 Conclusion

The presented theoretical results show, that the method of phase-shift keying of Bragg gratings allows flexible reconfigurations of an spectral filter and that it is very promising for the application as filter in DWDM systems and as part of reconfigurable multiplexers. A generalization of Kogelnik's theory of coupled waves has been used to find the appropriate phase-shift keying and to calculate the diffraction efficiency.

References

[1] H. Kogelnik, „Coupled wave theory for thick hologram gratings" in *Bell Syst. Tech. Jour.* **48**(9):2909+ (1969)

[2] C. Heinisch, S. Lichtenberg, V. Petrov, J. Petter, T. Tschudi, „Spectral Bragg filter with a synthesized transfer function" in *DGaO Proc.* (2005)