Aplanatism for positive and negative refractive index

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The development of optical metamaterials showed surprising variations of the imaging by optical surfaces between optical materials with different signs of the refractive index. In these cases, the stigmatic imaging Cartesian ovaloides contain a submanyfold of surfaces with astigmatic imaging. The aplanatism in Abbe's case with a virtual object is converted to a real imaging by a sphere

1 Introduction

The generalization of the Cartesian ovaloides [7] to optical surfaces separating media with different signs of refractive index [1,2,3] can be used as starting point for the investigation of the possibility of aplanatism [4,5,6] in such cases.

2 Cartesian ovaloides

The Cartesian ovaloides [7] for stigmatic imaging can be generalized to optical surfaces between media with different signs of the refractive index by demanding a constant optical path length

$$\sqrt{r^2+(a+z(r))^2}+n'\sqrt{r^2+(b-z(r))^2}=a+n'b$$

for the imaging problem shown in Fig. 1 with n = 1.

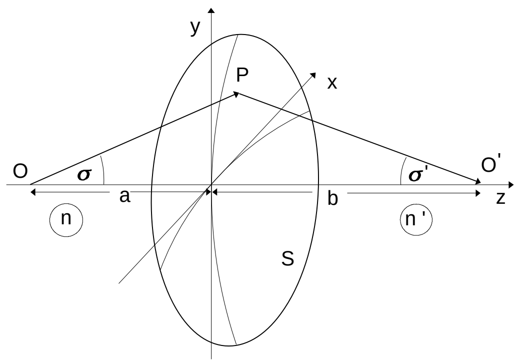
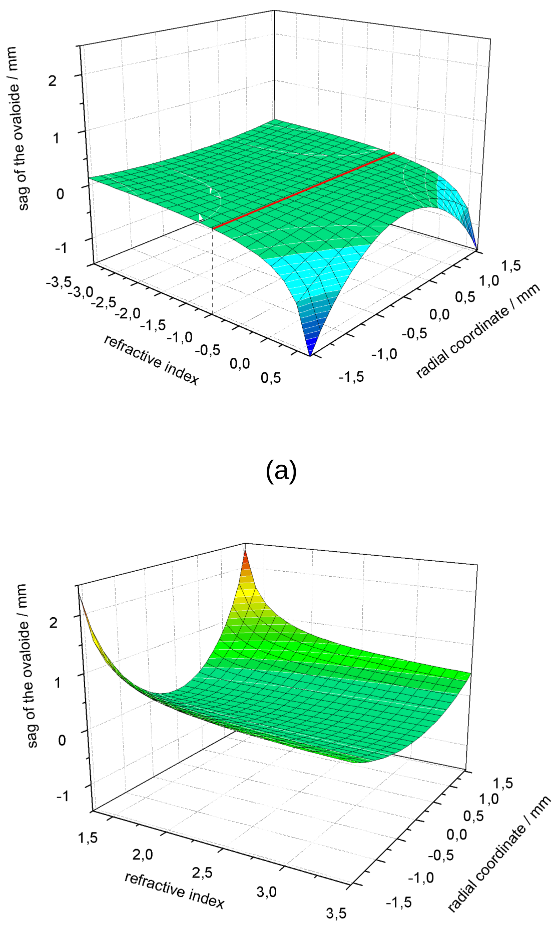


***Fig. 1** Real imaging OO' with help of the optical surface S.*

The sags of these rotational symmetrical surfaces are shown in Fig. 2 in dependence on the refractive index n'. The singular value n' = 1 is omitted. This case means that without refraction no imaging occurs. Therefore, Figs. 2 (a) and 2 (b) show two parts of the surface. The red marked straight line in Fig. 2 (a) shows that in the case of n' = -1 a plane is able to generate a stigmatic image [2].



(b)

***Fig. 2** Sags of Cartesian ovaloides for Fig. 1 (n = 1) in dependence on (a) n' < 0.7 (mostly negative) and (b) n' > 1.3 (positive).*

3 Aplanatism

The proposal of Pendry [2] includes a case of aplanatism: The quality of this stigmatic imaging does not change under a translation along a direction perpendicular to the normal vector of this surface.

The question to be answered is whether the Cartesian ovaloides show aplanatism for n’ < 0.

The answer can be found by comparing, in what cases Abbe’s sine condition [4]

$$\frac{n\,\sin(\sigma)}{n'\sin(\sigma')} = \frac{y_O'}{y_O} = \beta'$$

is compatible with the equation of the ovaloide

$$n\,\overline{OP} + n'\,\overline{PO'} = const.$$

The result [8] is the known configuration with two inversion points in relation to a circle, but with a real imaging for n’ < 0, where a virtual point is contained for n’ > 0.

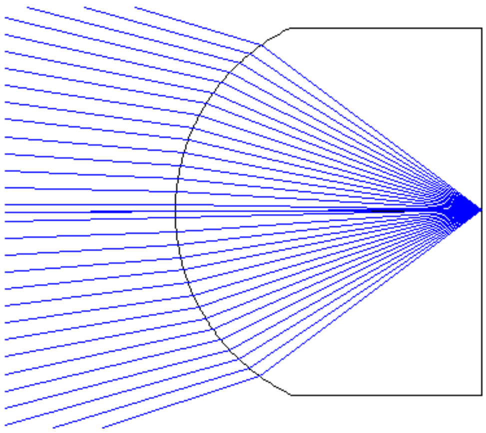


Fig. 3 *Abbe’s case of aplanatism for n = 1 and n’ = 2 > 0: virtual object.*

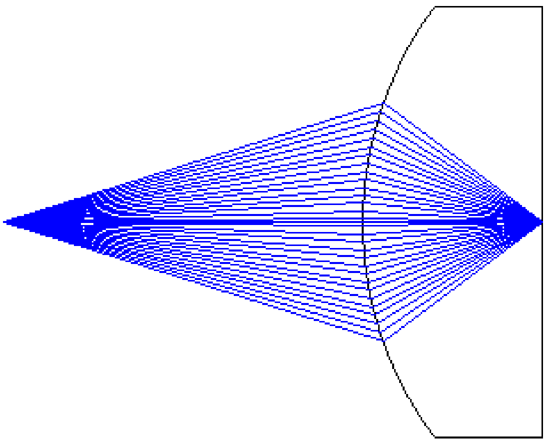


Fig. 4 *Aplanatism for n = 1 and n’ = -2 < 0: real object, real image.*

The essential difference between n’ > 0 and n’ < 0 occurs by a comparison of Fig. 3 with Fig. 4. In Abbe’s case, a real and a virtual point are connected by imaging, in the case n’ < 0 two real points occur.

In Fig. 5, the ray tracing is shown for the imaging of Fig. 4. The diameter of the spot diagram increases quadratically with the zone height of the image point, a property which confirms the aplanatism.

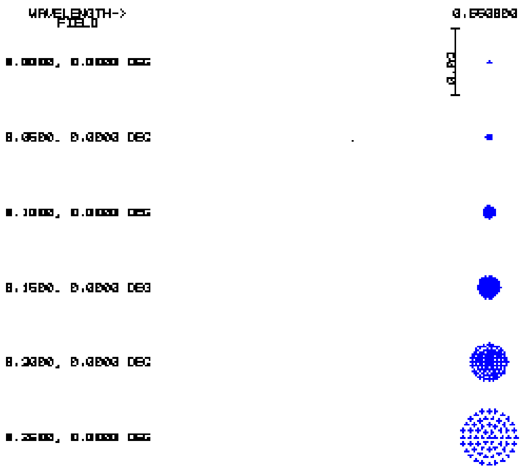


Fig. 5 *Spot diagram for increasing zone height of the imaging for n’ = -2 in Fig. 4.*

cases of Descartes ovaloides	real object point real image point	virtual object point real image point
second index positive	no aplanatism	aplanatism
Second index negative	aplanatism	no aplanatism

Tab. 1 *Different cases of aplanatsm for n > 0 with n’ > 0 and n’ < 0.*

Different cases of aplanatism are listed in Tab. 1 for real and virtual points. A more extended version is given in [8].

References

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