# Fast and robust 3D shape reconstruction from gradient data

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Sensors based on deflectometry measure the local slope of a surface. The object shape can then be reconstructed by spatial integration. Recently, we presented a method employing radial basis functions (RBFs) which accurately reconstructs both local details and the global shape. However, for large data sets the method requires a domain decomposition technique which may introduce error propagation. Further, it may be time consuming. Often the data is acquired on a regular grid. For this case, we present a method employing B-splines. It yields an accuracy similar to the RBF approach while being faster and more robust against error propagation.

## 1 Introduction

The increasing demand for optical free-form surfaces (see Fig. 1) has created new challenges for production and metrology. For industrial quality assurance, it is essential to acquire the 3D shape of the fabricated surface with high accuracy. That is to say, the measurement should be able to reproduce *both* the global shape of the object *and* local surface defects– the dynamic range between these two objectives possibly being as high as $1 : 10^6$.

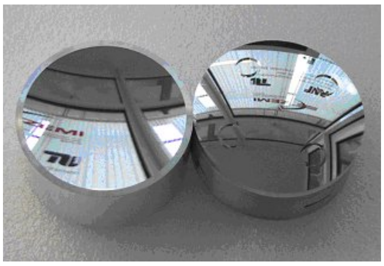

***Fig. 1*** *High-precision free form surfaces*

An optical 3D sensing method which is able to fulfill these requirements is *Phase-measuring Deflectometry* [1]. Deflectometric methods achieve high sensitivity by measuring not the shape of the surface but its local slope. This means that numerical integration is required to obtain the object's actual shape. Like the measurement itself, this numerical method should reproduce both the global shape and local details accurately, while not introducing any further artifacts.

We already introduced a method for numerical integration which is based on interpolation with radial basis functions (RBFs) [2]. The shape obtained by this method has the required accuracy [3]. However, for industrial applications like inline quality control this method is too time consuming. Therefore, a faster method is required.

Challenges to overcome :

- *A typical data set may contain up to two million data values*. The method should be able to handle them within a few seconds.
- *The measured slope data may be noisy*. The method should preserve the smoothing property of integration.
- *The grid is not necessarily complete*. The method should cope with missing data sites in the interior and with irregularly shaped borders.

We present a new method based on B-splines which reconstructs the height from measured gradients with improved accuracy in a shorter time. It is only applicable to the case of evenly spread data sets. However, this case applies to the majority of applications.

## 2 B-spline reconstruction method

Given the slopes of a surface, one possibility to obtain the shape is by *interpolation*: By matching the derivatives of an analytical interpolant with the slope data, the interpolant itself yields the desired surface representation.

$$\boxed{\begin{array}{rcl} \nabla(\text{interpolant}) & \overset{!}{=} & \text{slope data} \\ & \Downarrow & \\ \text{interpolant} & \overset{\wedge}{=} & \text{object shape} \end{array}}$$

The new method uses tensor-product B-splines as basis functions for the reconstruction. Similar to the

RBF interpolation approach, we need to solve a system of linear equations. With B-splines, however, the system matrix is *sparse*. This allows the application of efficient numerical solvers like *Conjugated Gradients* to reconstruct the surface in much less time and for bigger data sets.

Since there are two slope values given for each height value to be reconstructed, the system of equations is overdetermined in case of B-spline basis functions. Therefore, the new method handles this with an *approximation* approach: Instead of interpolating the slope values, the least-squares error between the measured and the reconstructed slopes is minimized.

The method is able to fill in missing data points in the interior of the measuring field. Although tensor-product B-splines require a rectangular grid, it is also possible to handle irregularly shaped boundaries by decomposing the data field into rectangular surface patches (see [4] for more details).

## 3 Results

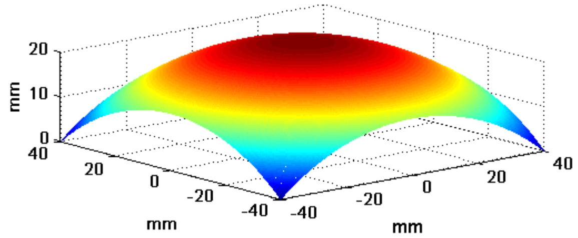

***Fig. 2*** *Reconstructed height of a simulated lens of radius 80 mm*

We simulated (under realistic conditions) the measurement of a spherical lens with radius 80 mm and sampling distance 0.2 mm on a measuring field of 80×80 mm$^2$, yielding 320,000 slope values (see Fig. 2). We added realistic noise of 8 arcsec to the sampled slope data and reconstructed the spherical surface both with the RBF and the B-spline method. With both methods, a decomposition of the data field into smaller patches is necessary to be able to handle the huge size of this data set. Using this decomposition, the reconstruction is performed on each patch separately and the patches are then "stitched" together again [3]. The B-spline approach greatly reduces the number of necessary patches (see Tab. 1). This also improves the accuracy, since the "stitching" process introduces error propagation (see Fig. 3).

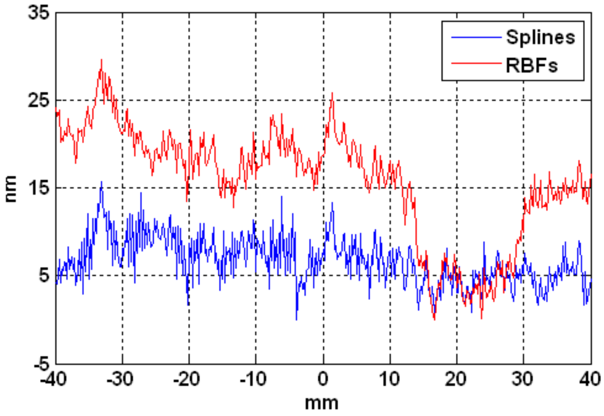

***Fig. 3*** *Absolute error of both methods (cross-section through the difference maps of the reconstructed spheres and the simulated sphere, in nm).*

| *property* | *RBFs* | *B-splines* |
|---|---|---|
| number of patches | 730 | 4 |
| max error | 70 nm | 30 nm |
| runtime | 62 sec | 18 sec |

***Tab. 1*** *Comparison of results, based on simulated data*

Table 2 shows a short summary, comparing the RBF method with the B-spline method. For medium-sized and partially complete measurements on a regular grid, both methods give good results. If the data sites are irregularly distributed, the B-spline method cannot be used and the RBF method should be employed instead. For large-area measurements on evenly spaced grids (which can easily be enforced by appropriate sampling) the B-spline method is superior both in time expense and accuracy.

| *conditions* | *RBFs* | *B-splines* |
|---|---|---|
| large-area meas. on a grid | 0 | ++ |
| incomplete meas. on a grid | + | + |
| irregularly distributed data | + | – |

***Tab. 2*** *Method recommendation*

### References

[1] M. C. Knauer, J. Kaminski, and G. Häusler, "Phase Measuring Deflectometry: a new approach to measure specular free-form surfaces," in *Proc. SPIE Nr. 5457*, pp. 366–376 (2004).

[2] S. Lowitzsch, J. Kaminski, M. C. Knauer, and G. Häusler, "Vision and Modeling of Specular Surfaces," in *Proc. VMV '05*, pp. 479–486 (2005).

[3] S. Ettl, J. Kaminski, M. C. Knauer, and G. Häusler, "Generalized Hermite interpolation with radial basis functions considering only gradient data," in *Curve and Surface Fitting: Avignon 2006*, pp. 141–149 (2007).

[4] E. Olesch, "3D-Oberflächenrekonstruktion aus gemessenen Neigungsdaten mittels B-Splines," Diploma thesis, Institute of Applied Mathematics, University Erlangen-Nuremberg (2007).