# Generation of second harmonic with non-diffraction limited radiation

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An analytical Gaussian-Schell-model based theory of partial coherent SHG determines the dependence of the frequency conversion on the focussing conditions and the $M^2$ in vertical and lateral dimensions, simultaneously. High $M^2$ diminishes the SHG.

## 1 Introduction

The direct second harmonic generation (SHG) by semiconductor lasers (e.g. [1]) was treated for the coherent case in [2,3]. Especially, in the high power cases there is a bending of the conversion efficiency off the full coherent case. Therefore, an analytical Gaussian-Schell-model based partial coherent theory of SHG is developed taking into account former work [5,6,7,8] and comparing with the experiment [8,9,10].

## 2 Assembly and formula

The derivation of the power conversion rate into the SH uses the nonlinear propagation kernel developed in [5] for the electrical field and for the partial coherence function [5-8]. The result is the conversion rate for the assembly given in Fig. 1:

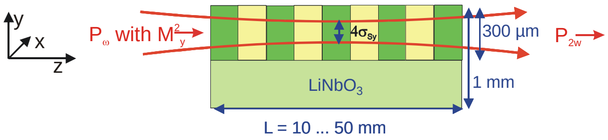

***Fig. 1*** *Assembly of SHG in a periodically poled lithium niobate crystal.*

$$P_{2\omega} \sim \sigma_{Sx}\sigma_{Sy}\sqrt{\left\{1+\left(\frac{z\lambda}{2\pi\sigma_{Sx}^2}\right)^2\right\}\{x \Rightarrow y\}}\int_0^l\int_0^l dz_1 dz_2 \, *$$

$$\left\langle\left\{\left[i\frac{2\pi}{\lambda}\sigma_{Sx}^2\left(f_1+f_2\right)\right]\left[-i\frac{2\pi}{\lambda}\sigma_{Sx}^2\left(f_3+f_4\right)\right]-\right.\right.$$

$$\left.\left.-z_1 z_2\left(\left\langle M_x^2\right\rangle^2-1\right)^2\right\}\{x \Rightarrow y\}\right\rangle^{-\frac{1}{2}}$$

with $$f_1=\left[1+(M_x^4-1)\right]\left[1+\frac{z_2}{z}\right]\left[\frac{z\lambda}{2\pi\sigma_{Sx}^2}\right]^2+4,$$

$$f_2=z_2\left[1+(M_x^4-1)\right],$$

$$f_3=\left[1+(M_x^4-1)\right]\left[1+\frac{z_1}{z}\right]\left[\frac{z\lambda}{2\pi\sigma_{Sx}^2}\right]^2+4,$$

$$f_4=z_1\left[1+(M_x^4-1)\right] \text{ and } z=-L/2.$$

with a factor, depending on the effective nonlinearity, the refractive indexes of the crystal, the velocity of light, the frequency of light and the free space permittivity.

The parameters in the formula are L ~ the crystal length, $\sigma_{Sx}$ and $\sigma_{Sy}$ ~ the half ellipse radii of the intensity beam waist along the x and y-direction respectively, $M_x^2$ and $M_y^2$ ~ the beam quality parameters along the x- and y-direction respectively. ($M_x^2$-1) is the deviation from the diffraction limit. {x→y} means to substitute x in the preceding expression by y.

## 3 Results for focusing and beam quality

If in the formula the beam quality factors are fixed for example to $M_x^2 = 3$ and $M_y^2 = 2$, then the SH power $P_{2\omega}$ depends on the shape of the intensity waist $\sigma_{Sx}$ and $\sigma_{Sy}$ as shown in Fig. 2. This provides an optimization tool for focusing with help of a beam guiding optics. The fixing of the focusing conditions for example by $\sigma_{Sx}$ = 20 µm and $\sigma_{Sy}$ = 20 µm results in a dependence of $P_{2\omega}$ on both of the beam quality parameters $M_x^2$ and $M_y^2$ as shown in Fig. 3. This surface describes the degradation of $P_{2\omega}$ by the increase of the beam quality. Fig. 4 shows an example for bending the quadratic dependence of $P_{2\omega}$ on $P_\omega$ caused by an quadratic increase of $M_x^2(P_\omega)$ in dependence on $P_\omega$ and $M_y^2$~ constant. This shows: The usual characterization of high beam quality lasers by the current-optical-power-characteristics and the conversion efficiency should be complemented by the curves $M_x^2(P_\omega)$ and $M_y^2(P_\omega)$ in cases of SHG applications. The experimental values of SHG from [8] are compared with the results from formula in Fig. 5. The placement parameter is the lateral beam diameter as abscissa.

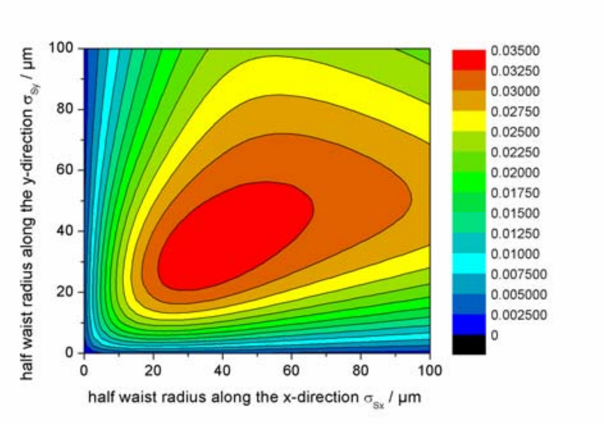

***Fig. 2*** *SH power $P_{2\omega}$ in dependence on the focusing conditions given by $\sigma_{Sx}$ and $\sigma_{Sy}$ at fixed beam quality factors $M_x^2 = 3$ and $M_y^2 = 2$.*

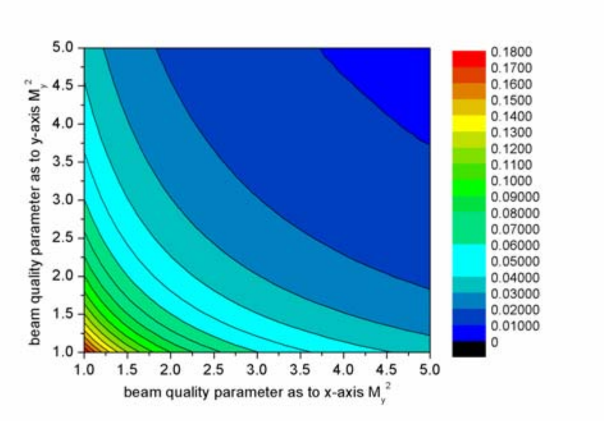

***Fig. 3*** *SH power $P_{2\omega}$ in dependence on the laser beam quality parameters $M_x^2$ and $M_y^2$ at fixed focusing parameters $\sigma_{Sx}$ = 20 µm and $\sigma_{Sy}$ = 20 µm.*

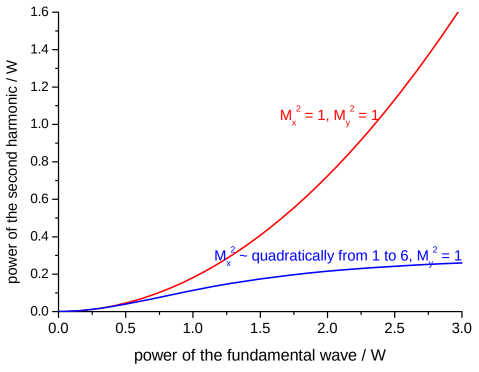

***Fig. 4*** *Bending of the quadratic characteristics of SHG by an increase of $M_x^2$ in dependence on $P_\omega$.*

## 4 Conclusion

The Gaussian-Schell-model based analytical calculations of the SHG connects demands for the optical beam guiding with the effects of the beam quality parameters. The addition of more parameters to the model of the light source (e.g. wave front aberrations) could increase the precision of the theory.

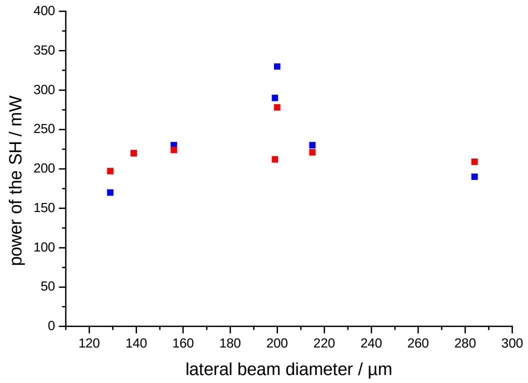

***Fig. 5*** *Comparison of experimental values from [8] with theory.*

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