Treatment of spatially varying Permeabilities with the RCWA - Application to Negative-Index Materials

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The standard implementations of the RCWA assume a permeability equal to one. With the increasing interest in meta-materials and negative-index materials, it is desirable, to simulate also geometries with spatially varying permeabilities. We have investigated the formulation of the RCWA for the two-dimensional and the three-dimensional case, including the Li-factorization and have extended the algorithms to include a spatially varying µ(x,y). The addition does not increase the computational complexity of the RCWA significantly.

1 Introduction

In recent times, there has been increasing interest in the creation and application of synthetic materials with optical parameters, which are not available in nature. In particular, negative index materials, predicted by Veselago [1] can now be fabricated for the near infrared and soon will be available also for the visible regime. Most prominent applications of these new materials include the super-lens, a flat substrate with $\varepsilon = -1, \mu = -1$ showing imaging performance, which is not limited by the numerical aperture. Another application is the optical cloak, an arrangement, capable of hiding objects in a spherical geometry, making it invisible to the outside.

The general strategy for realizing such materials is to fabricate sub-wavelength sized elementary cells, consisting of a combination of dielectrics and metals, which, according to the effective index theory, show unusual permittivities and permeabilities in a limited wavelength range.

The present way of simulating these structures rigorously is to model each elementary cell on the sub-wavelength scale and to construct larger structures as an assembly of these elementary cells. For a fully three-dimensional simulation with the RCWA, this method requires a large number of modes. Since the computational complexity of the 3D-RCWA grows with the sixth power of the number of modes, this method is limited in practice to relatively small structures.

2 The RCWA

The rigorous coupled wave analysis (RCWA) developed and improved by a series of authors [2,3] starts with the assumption, that the permeability is constant. Thus only one of the two Maxwell equations, establishing the basis for the RCWA contain the product with a spatially varying distribution. In the modal formulation this product is expressed as a multiplication with a Töplitz matrix implementing a convolution. In the reformulation of the RCWA for spatially varying ε, μ, these equations have to be reconsidered.

The general RCWA algorithm [2] can be decomposed into three steps: For each layer, an eigenvalue problem has to be solved, resulting in a vector of eigenvalues and a matrix of eigenvectors. The square root of the eigenvalues represents the z-component of the wave-vector for each mode. The second step is the Enhanced Transmisson Matrix Approach (ETMA) [2], which connects the incident field, the transmitted field and the field components at each layer as a function of the eigenvectors and eigenvalues of the layers. From the ETMA, the transmission and reflection coefficients can be calculated. If the complete field distribution is required, the third step is to apply the matrices obtained from the eigenvalue decomposition to the transmitted field.

The eigenvalue problem can be expressed either by the electric field or by the magnetic field. In the latter case, the eigenvalue problem is expressed as

$$\frac{\partial^2}{\partial z^2}\mathbf{U}_\perp - k_0^2 \mathbf{\Omega_1} \cdot \mathbf{U}_\perp = 0 \qquad (1)$$

The tangential field components at the interface between each layer are obtained from the propagation coefficients by

$$\begin{pmatrix} \mathbf{S}_\perp \\ \mathbf{U}_\perp \end{pmatrix} = \begin{pmatrix} -\mathbf{V\Phi}^-(z) & \mathbf{V\Phi}^+(z) \\ \mathbf{W\Phi}^-(z) & \mathbf{W\Phi}^+(z) \end{pmatrix} \begin{pmatrix} \vec{C}^+ \\ \vec{C}^- \end{pmatrix} \qquad (2)$$

with $\mathbf{W}$ as the matrix of eigenvectors and

$$\mathbf{V} = \mathbf{\Omega}_2^{-1}\mathbf{W}\mathbf{Q} \tag{3}$$

Compared to the standard formulation, only the two matrices $\mathbf{\Omega}_1$ and $\mathbf{\Omega}_2$ are affected by the introduction of a spatially varying permeability. For the standard formulation $\mathbf{\Omega}_1$ is given by

$$\begin{pmatrix} \mathbf{E}_{ay}^{-1}\mathbf{K}_y\mathbf{E}^{-1}\mathbf{K}_y - \mathbf{E}_{ay}^{-1} + \mathbf{K}_x^2 & \mathbf{K}_x\mathbf{K}_y - \mathbf{E}_{ay}^{-1}\mathbf{K}_y\mathbf{E}^{-1}\mathbf{K}_x \\ \mathbf{K}_y\mathbf{K}_x - \mathbf{E}_{ax}^{-1}\mathbf{K}_x\mathbf{E}^{-1}\mathbf{K}_y & \mathbf{E}_{ax}^{-1}\mathbf{K}_x\mathbf{E}^{-1}\mathbf{K}_x - \mathbf{E}_{ax}^{-1} + \mathbf{K}_y^2 \end{pmatrix}$$

In the variable µ case, the four matrix elements of $\mathbf{\Omega}_1$ in the order M_{11}, M_{12}, M_{21}, M_{22} are given by:

$$\mathbf{E}_{ay}^{-1}\mathbf{K}_y\mathbf{E}^{-1}\mathbf{K}_y - \mathbf{E}_{ay}^{-1}\mathbf{M}_{ay}^{-1} + \mathbf{K}_x\mathbf{M}^{-1}\mathbf{K}_x\mathbf{M}_{ax}^{-1}$$

$$\mathbf{K}_x\mathbf{M}^{-1}\mathbf{K}_y\mathbf{M}_{ax}^{-1} - \mathbf{E}_{ay}^{-1}\mathbf{K}_y\mathbf{E}^{-1}\mathbf{K}_x$$

$$\mathbf{K}_y\mathbf{M}^{-1}\mathbf{K}_x\mathbf{M}_{ay}^{-1} - \mathbf{E}_{ax}^{-1}\mathbf{K}_x\mathbf{E}^{-1}\mathbf{K}_y$$

$$\mathbf{E}_{ax}^{-1}\mathbf{K}_x\mathbf{E}^{-1}\mathbf{K}_x - \mathbf{E}_{ax}^{-1}\mathbf{M}_{ay}^{-1} + \mathbf{K}_y\mathbf{M}^{-1}\mathbf{K}_y\mathbf{M}_{ay}^{-1} \tag{4}$$

The $\mathbf{E}$-matrices are the familiar Töplitz matrices for the permittivity. $\mathbf{E}_{ax}$ and $\mathbf{E}_{ay}$ represent the Li-factorized versions of $\lfloor\lceil\varepsilon\rceil\rfloor$ and $\lceil\lfloor\varepsilon\rfloor\rceil$. The $\mathbf{M}$-matrices are the new counterparts for the permeability. The matrix $\mathbf{\Omega}_2$ in the standard formulation is given by:

$$\mathbf{\Omega}_2 = \begin{pmatrix} -\mathbf{K}_x\mathbf{K}_y & -\left(\mathbf{E}_{ay}^{-1} - \mathbf{K}_x^2\right) \\ \mathbf{E}_{ax}^{-1} - \mathbf{K}_y^2 & \mathbf{K}_y\mathbf{K}_x \end{pmatrix} \tag{5}$$

whereas in the new formulation it is given by

$$\mathbf{\Omega}_2 = \begin{pmatrix} -\mathbf{K}_x\mathbf{M}^{-1}\mathbf{K}_y & -\left(\mathbf{E}_{ay}^{-1} - \mathbf{K}_x\mathbf{M}^{-1}\mathbf{K}_x\right) \\ \mathbf{E}_{ax}^{-1} - \mathbf{K}_y\mathbf{M}^{-1}\mathbf{K}_y & \mathbf{K}_y\mathbf{M}^{-1}\mathbf{K}_x \end{pmatrix} \tag{6}$$

With only these two changes, the RCWA can be applied in the same way as for the constant permeability case.

3 Verification

In order to verify the numerical results, we have used the plane wave decomposition in multi-layer media [4]. For each plane wave component, the algorithm applies the standard multi-layer matrix theory, found in many text-books. The comparison shows not only a perfect agreement for the transmission and reflection coefficients but also a perfect agreement for the E- and H-field inside and outside of the layers.

4 Examples

In the first example (fig. 1), we have modelled a super-lens as a sequence of air, a negative index slab and air. We treated both, the case of perfect and imperfect negative index. Even for $\varepsilon = -1.3$, $\mu = -0.6$, a clear image of the focal distribution was visible, but slightly shifted in position.

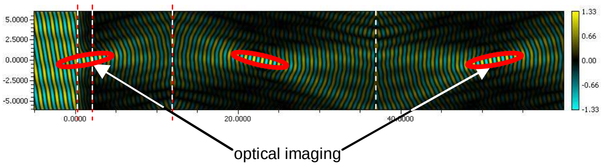

Fig. 1 *Perfect imaging through a slab of negative index material.*

In the second example, we modeled the field distribution for a blazed grating, where the blaze material has negative index.

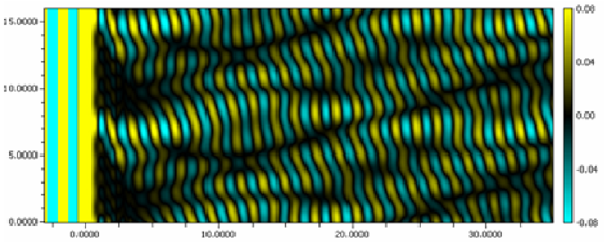

Fig. 2 *Field distribution around a blazed grating with negative index material. The wave fronts are tilted in the opposite direction compared to ordinary blazed gratings.*

Figure 2 shows a strong reflection and wave fronts which are tilted upward, as compared to ordinary blazed gratings, where the wave-fronts are tilted downward.

5 Conclusion

We have extended the standard RCWA to include also spatially varying permeabilities. The changes compared to the standard RCWA only affect two matrices. The new formulation can serve as a tool for investigation of structured left-handed materials.

References

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