# Validity of scalar efficiency approximations for stray light estimation of DOEs for color correction

Bernd H. Kleemann and Markus Seesselberg

*Carl Zeiss AG, Corporate Research and Technology, D-73446 Oberkochen, Germany*

*mailto:b.kleemann@zeiss.de*

The validity of scalar efficiency approximations based on the thin element approximation is discussed. These formulae enable optical designers to estimate the amount of stray light from unwanted diffraction orders. An accurate rigorous electromagnetic integral method is used as a benchmark.

## 1 Introduction

Diffractive optical elements (DOE) are well–suited for the correction of longitudinal and transverse chromatic aberrations in head mounted displays, camera lenses, infrared optics and other broadband optical systems. Unfortunately, the diffraction efficieny $\eta$ for the working order of DOEs is clearly below 100 % which causes stray light from unwanted diffraction orders. Main reasons for the decrease of the diffraction efficiency $\eta$ are spectral bandwidth, variation of incidence angles and manufacturing errors.

Known scalar approximations for the diffraction efficiency are collocated and their limits are inspected by comparing with electromagnetic calculations as a benchmark, i. e. by solving Helmholtz's equations. Unpolarised efficiency is calculated to be comparable with scalar values. Often, simple scalar formulae approximate the diffraction efficiency to good accuracy.

Following assumptions are made. The first diffraction order with $m = 1$ is the working order of the DOE. The local grating assumption is used, i. e. the diffraction efficiency is calculated for an infinitely large grating with constant grating period. Since DOEs for color correction in broad band optical systems are investigated, the groove width $d$ is much larger than the wavelength $\lambda$. Diffraction efficiencies $\eta$ are given in percentage from transmitted light. In other words, reflected light is not considered here as it does not directly influence the image and can be reduced by e. g. antireflection coatings.

A more detailed elaboration of the subject of this contribution with efficiency curves of all considered dependencies is given in [1].

## 2 Efficiency as a function of wavelength

Let us consider a kinoform DOE with perfect Fresnel–looking geometry which is intended to be used in an optical system designed for a spectrum ranging from $\lambda_{\mathrm{short}}$ to $\lambda_{\mathrm{long}}$. The wavelength $\lambda_0$ describes an intermediate wavelength where the diffraction efficiency should reach its maximum. Then, the groove depth $h_0$ of the kinoform obeys $h_0 = \lambda_0/[n_1(\lambda_0) - n_2(\lambda_0)]$ where $n_1(\lambda) > n_2(\lambda)$ without loss of generality. Based on the thin element approximation (TEA) and for normal incidence of light to the DOE, the efficiency of order $m$ is given by

$$\eta_m(\lambda) = \mathrm{sinc}^2\left(\frac{h_0}{\lambda}\Big(n_1(\lambda) - n_2(\lambda)\Big) - m\right) \qquad (1)$$

with $\mathrm{sinc}(x) \equiv \sin(\pi x)/(\pi x)$.

## 3 Efficiency as a function of wavelength and angle of incidence

The diffraction efficiency $\eta_1$ also depends on the angle of incidence $\theta$ of light incident on the DOE in a plane perpendicular to the grooves of depth $h_0$. $\theta$ is given in the medium with refractive index $n_2(\lambda)$ (usually air). Again, using TEA, we derive for the diffraction efficiency of diffraction order $m$

$$\eta_m(\lambda, \theta) = \mathrm{sinc}^2\Bigg\{ \qquad (2)$$

$$\frac{h_0}{\lambda}\left[n_1(\lambda)\cos\theta - \sqrt{n_2(\lambda)^2 - n_1(\lambda)^2 \sin^2\theta}\right] - m\Bigg\} \quad . \qquad (3)$$

The above equation also holds for poor conical incidence, i. e. when the light is incident on the DOE in a plane parallel to the grooves (cf. [2, Fig. 6]).

## 4 Efficiency as a function of fabrication errors

Now we consider a DOE which is fabricated with correct blaze angle $\alpha_0$, i. e. the ascent angle of the DOE facets (cf. Fig. 1).

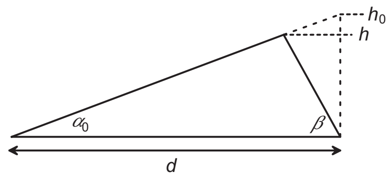


***Fig. 1*** *A groove with local zone width $d$, blaze angle $\alpha_0$ and antiblaze angle $\beta$. For an ideal groove, the antiblaze angle is $\beta = 90^\circ$. Due to manufacturing errors, $\beta$ does not achieve its design value and the actual groove depth $h$ is below the design groove depth $h_0$.*

Due to fabrication errors occuring e.g. when the DOE is fabricated by diamond turning, the DOE facet is often too short leading to an actual groove depth $h$ which is smaller than the design groove depth $h_0$. Indeed, this typical fabrication error leads to a pronounced decrease in diffraction efficiency especially in case of a small groove width $d$, e. g. $d \approx 15\,\mu$m.

The diffraction efficiency of order $m = 1$ for normal incidence and wavelength $\lambda_0$ as a function of $h$ is given by

$$\eta_1(\lambda_0, h) = \left(\frac{n_1(\lambda_0) - n_2(\lambda_0)}{\lambda_0} \cdot h\right)^2 = \left(\frac{h}{h_0}\right)^2 \quad (4)$$

with groove depth $h_0$ as given in Sect. 2. Simple trigonometrical considerations show the equivalence of this expression to

$$\eta_1(\lambda_0, d, \beta) = \left(\frac{d \tan\beta}{h_0 + d \tan\beta}\right)^2 \quad (5)$$

which now depends on local zone width $d$ and antiblaze angle $\beta$. Often, (5) is more handy from a practical point of view since the antiblaze angle $\beta$ is constant for a DOE due to manufacturing reasons. Thus the diffraction efficiency for wavelength $\lambda_0$ and normal incidence depends only on zone width $d$.

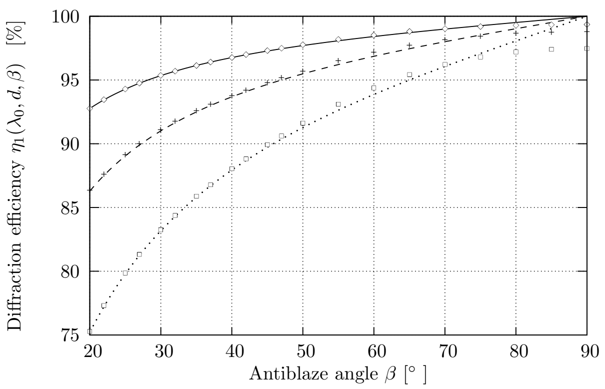


***Fig. 2*** *Diffraction efficiency $\eta_1(\lambda_0, d, \beta)$ as a function of antiblaze angle $\beta$ for normal incidence according to equation (5) for the blaze angle which is appropriate for design wavelength $\lambda_0 = 550$ nm. Solid (dashed, dotted) lines correspond to zone width $d = 80\,\mu$m ($d = 40\,\mu$m, $d = 20\,\mu$m). Marks are results obtained by electromagnetic simulation for the design wavelength and zone widths $d = 80\,\mu$m (diamonds), $d = 40\,\mu$m (crosses), and $d = 20\,\mu$m (squares). Material 1 (2) is PMMA (air).*

For a DOE made from the materials PMMA and air, the diffraction efficiency $\eta_1(\lambda_0 = 550\,\text{nm}, d, \beta)$ as a function of antiblaze angle $\beta$ for different values of zone width $d$ according to expression (5) is depicted in Fig. 2. The increase of diffraction efficiency with zone width $d$ for a constant antiblaze angle $\beta$ is clearly seen. Since the zone width $d$ usually decreases with radial distance $r$ from the center of the DOE, the diffraction efficiency decreases with $r$ as well.

## 5 Efficiency as a function of surface roughness

Considering surface roughness, one has to distinguish between a statistical micro–roughness of an etching process and a probably periodical micro-roughness introduced by machining tools, for instance by the diamond tool of a single point diamond turning process. Dependency of diffraction efficiency from statistical micro roughness $R_q$ leads to the Debye-Waller factor

$$\eta_1(\lambda, \theta_2, R_q) = \eta_1(\lambda, \theta_2) \cdot \left[1 - \left(\frac{2\pi R_q}{\lambda}\Big(n_1(\lambda) - n_2(\lambda)\Big)\right)^2\right] \quad (6)$$

where $\eta_1(\lambda, \theta_2)$ denotes the efficiency without roughness according to equation (2) and $R_q$ is the RMS–value of the profile roughness. For a statistical surface roughness without periodicity, stray light has a broad angular distribution and is not directed in unwanted diffraction orders.

Also for periodical micro–roughness on the blaze facets using electromagnetic simulations, the amount of stray light is predicted by equation (6). It is important to note that equation (6) does not depend on the specific shape of roughness which can be a sinusoidal, triangular or binary profile. Then $R_q$ is simply the RMS–value of the amplitude of the periodical roughness. The only difference between periodical and statistical micro–roughness lies in the distribution of stray light to various diffraction orders. Depending on wavelength, incidence angle and period of roughness, the stray light from unwanted diffraction orders is concentrated in one or two diffraction orders with $m \neq 1$.

### References


[1] M. Seesselberg and B. H. Kleemann, “DOEs for Color Correction in Broad Band Optical Systems: Validity and Limits of Efficiency Approximations,” in *OSA International Optical Design Conference (IODC)*, Proc. SPIE (2010). IThB5: accepted for publication.

[2] G. Schmidt and B. H. Kleemann, “Integral equation methods from grating theory to photonics: An overview and new approaches for conical diffraction,” J. mod. Opt. (2010). Accepted for publication.