Lightfield Photography and the Tomographic Analysis of Optical Phase Space

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A heuristic interpretation of the optical instrument function is developed based on the framework of the Wigner distribution function. An alternative derivation of phase-space tomography is presented which provides a link between lightfield imaging and the phase space of coherent and partially coherent signals.

1 Introduction

New optical hardware, such as high resolution detectors, provide the technological basis for a rapidly evolving imaging technology. In this context, the well-known concept of Lippmann's integral photography is reintroduced as part of lightfield photography [1]. The optical lightfield can be identified as the phase-space of geometrical optics, and the captured output signal is identical to the radiance distribution of the optical signal.

Some recent work has recognized the importance of extending the lightfield description to model coherent and partially coherent wave propagation [2]. Treatment of complex amplitudes and mutual coherence functions typically requires additional mathematical tools, such as the Wigner distribution function (WDF),

$$W(x,\nu) = \int_{\infty}^{\infty} u(x+x'/2)u^*(x-x'/2)e^{-i2\pi\nu x'}dx', \quad (1)$$

where $u(x)$ is the complex amplitude.

In this work, I demonstrate that the optical hardware of lightfield photography can be used to reformulate the concept of phase-space tomography [3]. Lightfield photography is identified as a special form of phase-space tomography applicable whenever the optical signal can be modeled with ray optics.

2 The optical instrument function

A typical way to use phase-space optics is the translation of signals and systems into the respective WDFs and to apply the operations of optical system theory, i.e. modulations and linear input-output systems, in phase space.

A less well-known approach splits the entire optical system into one part which transforms the complex amplitude, and a second part, which includes the square wave detector at the output. The latter probes the optical wavefield and can be interpreted as a generalized detector. Each detector element attached to the generalized detector is considered as a single instrument and the function characterizing this device is called the instrument function [4], which is detecting optical radiation over a certain area and solid angle. The instrument can equally be described with a phase-space distribution and the detected power is the overlap integral between the signal WDF $W_s(x,\nu)$ and the detector WDF $W_d(x,\nu)$,

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_s(x,\nu)W_d(x,\nu)d\nu dx \quad (2)$$

The concept was applied previously to obtain a phase-space description for the power coupling into optical waveguides [5]. In Fig. 1(a), a monomode waveguide attached to a power detector can be associated with the instrument and the corresponding WDF is computed from the guiding mode.

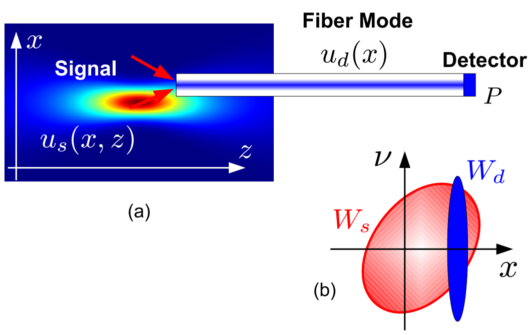

Fig. 1 *The overlap integral in phase space: (a) optical fiber sensor probing the coherent wavefield; (b) phase-space diagram of the overlap integral specifying the detect output power.*

3 Constructing the instrument function

For optical applications, the instrument function can be constructed based on heuristic arguments. In particular, it is possible to derive the instrument WDF by backpropagating the WDF of the square law point detector element to the input plane of the instrument. If the instrument consists of a microlens with a detector located in its focal plane (Fig. 2), the WDF of a point detector $W_d(x,\nu) = \delta(x - x_d)$ has to be propagated to the lens, followed by chirping with the inverse lens function, and finally modulated by the

WDF of the lens aperture. The resulting WDF the corresponds to the WDF of a rectangle shifted in frequency (Fig 2(b)). To take into account the finite size a of the power detector, the detector is treated as an incoherent signal, i.e. $W_d(x,\nu) = \text{rect}[(x - x_d)/a]$, which is then backpropagated to the detection plane.

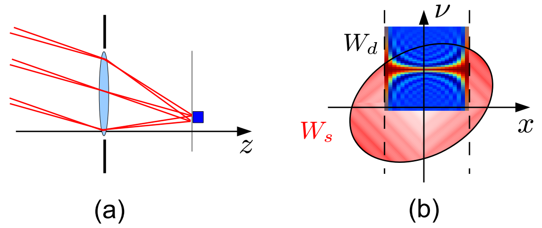

Fig. 2 *The instrument function in phase space: (a) optical instrument (point detector in the focal plane of a microlens), and (b) phase-space diagram.*

4 Alternative formulation of phase-space tomography

The instrument function permits one to reformulate phase-space tomography. Instrumental to this approach is the ambiguity function (AF) which is introduced here as the two-dimensional Fourier transform of the WDF,

$$A(\bar{\nu}, \bar{x}) = \int\int W(x,\nu) e^{-i2\pi(x\bar{\nu} - \nu\bar{x})} dx\, d\nu \qquad (3)$$

Parseval's theorem ensures that in the AF domain the overlap integral again takes the form of Eq. (2). We again consider a microlens/point detector device, where the instrument function is schematically associated with the elliptical domain in Fig. 3(a). The corresponding phase-space volume in the AF domain is characterized by reciprocal extensions in $\bar{\nu}$ and $\bar{x}$. This results in a measurement corresponding to the signal AF averaged over the phase space footprint of the instrument. To resolve individual points $(\bar{\nu}_0, \bar{x}_0)$ within the volume defined by the instrument either the entire instrument (Fig. 3(b)) or the focal plane detector (Fig. 3(c)) can be moved. The corresponding projector in the AF domain is the original instrument function modulated with a linear phase factor in $\bar{\nu}$ and $\bar{x}$, respectively. From this set of measurements the product of signal AF and known instrument AF can be recovered from an inverse Fourier transformation.

If the signal AF is essentially limited to the AF volume of the instrument, sampling phase space with the lenslet array and the focal plane detector array of lightfield photography will be sufficient for recovering the entire phase space information. This corresponds to the geometrical optics approximation assumed for lightfield photography. If the AF needs to be recovered outside the volume of the original instrument function measurements with larger and smaller lens apertures need to be added to extend the total AF coverage. This step also provides the conceptual link between lightfield photography and phase-space tomography.

It also suggests a phase-space approach for constructing imaging and sensing systems in general. Depending on the typical phase-space volume occupied by the signal, a minimum set of instruments can be identified (e.g. characterized by the instrument aperture and the lateral position of lens and detector), which allows one to recover the signal.

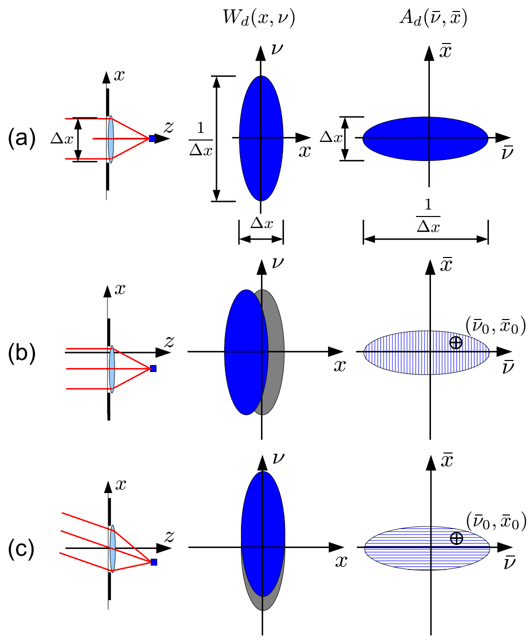

Fig. 3 *Phase-space tomography with microlens sensor: (a) Optical instrument, schematic WDF, and AF; (b) lateral shift of the sensor corresponding to a shift in x of the WDF and a modulation of the AF in $\bar{\nu}$; (c) lateral shift of the focal plane detector corresponding to a shift in ν of the WDF and a modulation of the AF in $\bar{x}$.*

References

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