Precise measurement of internal transmittance of high absorbing filter glass

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The calculation of the internal transmittance for optical filter glasses can be performed by using transmittance data originated from two different sample thicknesses. However, the amount of absorption has to be considered. Therefore, the sample thicknesses have to be chosen carefully in order to achieve accurate results. Typical measurement uncertainties are discussed and typical examples are presented. Finally recommendations for suitable sample thicknesses are given.

1 Definitions

a = P_1/P_2	ratio of the reflection factors P_1/P_2
d	thickness of the glass
τ	transmittance
τ_i	internal transmittance
$P = \tau/\tau_i$	reflection factor

2 Introduction

The internal transmittance of a glass cannot be measured directly. Instead the internal transmittance is calculated based on the measurement of transmittance and/or reflectance. However, the internal transmittance is of great importance during a design process of optical systems. Especially, the correct dimensioning of an absorption filter component relies on the exact knowledge of the spectral internal transmittance. Due to the low transmittance in the spectral region of absorption of a thick filter, there is usually no reliable measurement for the transmittance at the desired thickness.

The following paragraphs give the well known equations for the correct calculation of internal transmittance [1, 2]. Additionally, we give some advice for a good choice of sample thickness in order to achieve most accurate results.

3 Equations for internal transmittance

The internal transmittance can be calculated by the results of transmittance measurements of two samples which have different thicknesses via the following equations. The internal transmittance is derived for a thickness of d3 with d3 = d1 - d2. The transmittance τ_1 is the result of the measurement at thickness d1, and the transmittance τ_2 is measured at thickeness d2 of the filter glass.

There are two cases and the internal transmittance τ_{i3} is given by:

a) High values of both transmittance τ_1 and τ_1:

$$\tau_{i3} = \frac{\tau_1}{\tau_2} \tag{1}$$

b) Very different values of transmittance τ_1 und τ_1 (region of strong absorption):

$$\tau_{i3} = \frac{\tau_1}{\mathrm{a}\,\tau_2} \tag{2}$$

4 Basic equations

The relation between transmittance and internal transmittace is [2]:

$$\tau_i = \frac{\tau}{P} \tag{3}$$

For any given thickness d the internal transmittance $\tau_{i\,d}$ can be derived from the internal transmittance $\tau_{i\,ref}$ at a reference thickness d_{ref} [2]:

$$\tau_{i\,d} = \left(\tau_{i\,ref}\right)^{d/d_{ref}} \tag{4}$$

For any measurements of transmittance for two samples of different thickness we get:

$$\tau_{i1} = \frac{\tau_1}{P_1} \qquad \text{and} \qquad \tau_{i2} = \frac{\tau_2}{P_2}$$

5 Spectral region of high transmittance

In case of high transmittance we assume

$$P_1 = P_2 = P \tag{5}$$

Using the equations 3,4 and 5 we achieve:

$$\left[\frac{\tau_1}{\tau_2^{\left(\frac{d_1}{d_2}\right)}}\right]^{\frac{d_2}{d_2-d_1}} = \mathrm{P} \tag{6}$$

An internal transmittance τ_{i3} at thickness

$$d_3 = d_1 - d_2 \tag{7}$$

is according to equation (3)

$$\tau_{i3} = \left(\frac{\tau_1}{\mathrm{P}}\right)^{\left(\frac{d_1-d_2}{d_1}\right)}$$

Now we eliminate P using equation 6 and finally we get

$$\tau_{i3} = \frac{\tau_1}{\tau_2}$$

6 Spectral region of low transmittance

In case of a spectral region with high absorption, equation (5) is no longer valid and instead we have to use the measurement of reflection and write

$$P_1 = \mathrm{a}\ P_2 \qquad (8)$$

Using the same scheme as before, equation 6 changes into:

$$\mathrm{P}_1 = \mathrm{a}\ \mathrm{P}_2 = \mathrm{a}\left[\frac{\tau_1}{\mathrm{a}\ \tau_2^{\left(\frac{d_1}{d_2}\right)}}\right]^{\frac{d_2}{d_2 - d_1}} \qquad (9)$$

And this gives finally equation 2:

$$\tau_{i3} = \frac{\tau_1}{\mathrm{a}\ \tau_2}$$

7 Effect of measurement uncertainties

In real life one needs to calculate the internal transmittance at a reference thickness d_{ref}. However, the measured values for transmittance were obtained at the thicknesses d1 and d2. Using equations (1) and (4) lead to an internal transmittance $\tau_{i,ref}$:

$$\tau_{i,ref} = \left(\frac{\tau_1}{\tau_2}\right)^{\frac{d_{ref}}{d_1 - d_2}} \qquad (10)$$

There is a uncertainty of measurement on the values for transmittance as well as for the values of the thicknesses:

Uncertainty of transmittance = $\pm\Delta\tau$ (11)

Uncertainty of thickness = $\pm\Delta d$ (12)

This yields a total uncertainty $\Delta\tau_{i,ref}$ for the calculated internal transmittance at the reference thickness of

$$\Delta\tau_{i,ref} = \tau_{i,ref,\max} - \tau_{i,ref,\min}$$

$$= \left(\frac{\tau_1 + \Delta\tau}{\tau_2 - \Delta\tau}\right)^{\frac{d_{ref}}{d_1 - d_2 + 2\Delta d}} - \left(\frac{\tau_1 - \Delta\tau}{\tau_2 + \Delta\tau}\right)^{\frac{d_{ref}}{d_1 - d_2 - 2\Delta d}} \qquad (13)$$

This uncertainty has its minimum when there are two boundaries satisfied:

- First, choose d2 as small as possible
- Second, the best choice of d1 is dependent on the amount of light that can still permeate the filter glass.

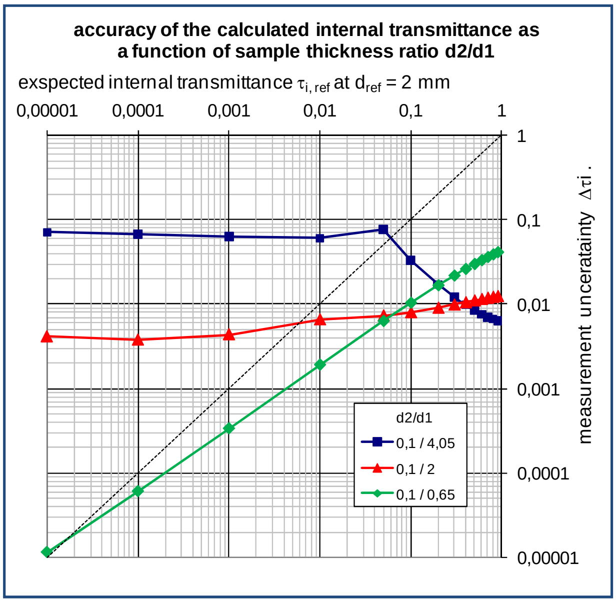

Fig. 1 *relation between accuracy and expected transmittance. The calculation was performed for an uncertainty of measurement for thickness of 0.002 mm and an uncertainty for the transmittance of 0.003.*

Figure 1 depicts three sets of choices for the sample thicknesses d1 and d2. The resulting uncertainty of the calculated transmittance is plotted over the internal transmittance of a filter glass. So for measuring a transmittance of 0.8 the best choice is d2 = 0.1 mm and d1 = 4.05mm (blue graph). However, for a low transmittance, let's say 0.001 the thickness d1 should be 0.65 mm and d2 =0.1mm, in order to give the lowest uncertainty of the measurement (green graph).

References:

[1] M. Born and E. Wolf: Principles of Optics (7th edition), Cambridge University Press, Cambridge 1999

[2] Reichel, S. and Biertümpfel, R.: Choosing the Correct Optical Filter for Your Application, Short Course SC1013 Photonics West 2011, San Francisco