

# Numerical Propagation Algorithms and Phase Retrieval Techniques

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Phase retrieval is an iterative technique for estimating the complex amplitude of a wavefield. Here several factors that affect the convergence of the algorithm are examined using real experimental results. A filter is proposed that increases the robustness of the algorithm.

## 1 Introduction

Phase Retrieval (PR) techniques represent an alternative to holography for estimating the complex amplitude of an optical wavefield. Usually at least 2 intensity measurements at longitudinally displaced optical planes are used so that the iterative Gerchberg-Saxton-type algorithm converges [3]. It is expected that multiple ( $>2$ ) intensity measurements from different planes will improve the resulting phase profile estimate [1, 2, 4, 5]. Other factors also play a role, for instance the initial guess at starting phase distribution (see Flow Chart at Fig. 2) is often chosen to be random, although other distributions may produce better convergence. The choice of numerical propagation algorithm is also important as well control of the frequency extent of the signal. Short distances between intensity measurements mean that the Spectral Method (SM) is a better choice than the Direct Method [6].

Here a preliminary investigation into the performance of a multi-plane PR scheme for real experimental results is performed. Best results were obtained by filtering in the spatial frequency domain as the SM was implemented.

## 2 Experimental Setup

Fig. 1 shows the experimental setup. The collimated laser beam (wavelength of 633nm) is spatially filtered with a pinhole and expanded before it illuminates a transparent microchannel volume. A telescopic optical system used to magnify the object before it is captured at the camera plane.

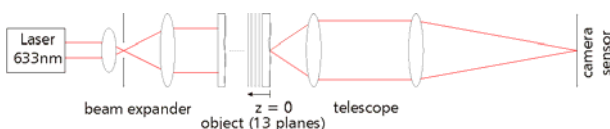


Fig. 1 The optical setup.

This experimental setup was used to capture 13 different intensity images of a volume containing scattering particles. Between captures the volume

was displaced longitudinally in steps of 50  $\mu\text{m}$ . Images 1, 3, 5 etc. are used in the PR algorithm, while Images 2, 4, 6 etc. are used as a “Gold standard image” to compare against the predictions of the phase retrieval algorithm.

## 3 Algorithm

The basic method is shown in Fig. 2. It is derived from a Gerchberg-Saxton iteration algorithm using multiple planes. Starting with a guess of a complex object wave composed of an initial arbitrary phase and the measured amplitude at plane 1, this distribution is propagated forward to the subsequent plane. Following [2] the amplitude is replaced by the multiplication with the measured amplitude at plane 2 being the next starting point for the alignment/propagation to the following plane until the last plane N is reached. Then, a back propagation to the first plane is performed and the next iteration cycle is started.

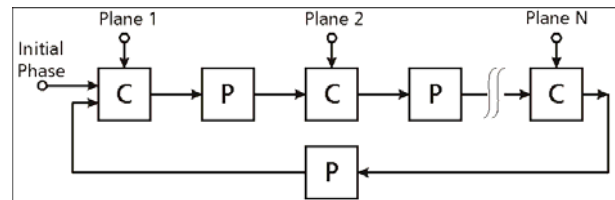


Fig. 2 Flow chart of algorithm.

The principle of the combiner is depicted in Fig 3 (left). The first possibility to reduce high frequency noise is the low-pass filtering of the measured amplitude before the alignment.

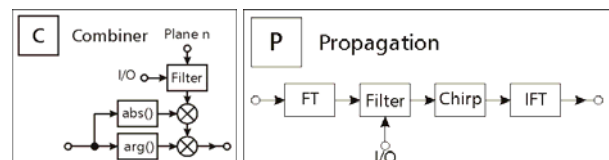
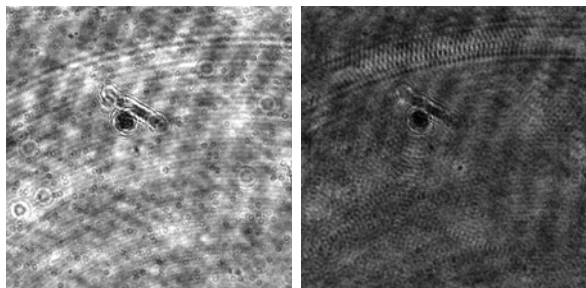


Fig. 3 Flow chart of combiner (left) and propagation (right).

The modified propagation part is shown in Fig. 3 (right). Using the SM a low-pass filtering can be carried out before the multiplication of the spectral amplitude with the chirp propagator.

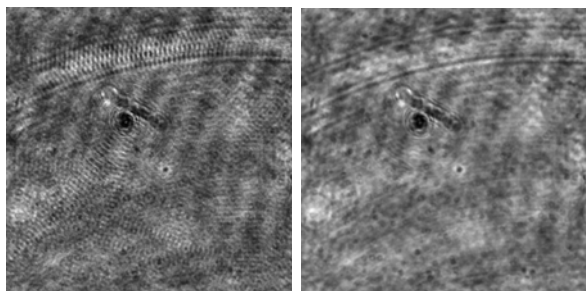
## 4 Experimental Results

The experimental results are evaluated at plane 2 (Image 2).



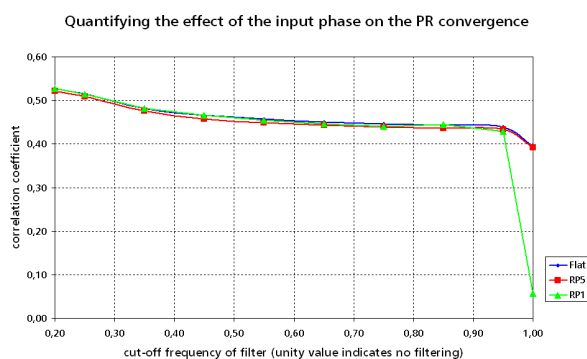
**Fig. 4** Actual experimental result of Image 2 (left), estimated intensity distribution at the same plane using PR technique (right).

Fig. 4 (left) represents the actual experimental result for  $z = 50$  microns. An estimated intensity distribution at the same plane using PR technique without any filtering is shown in Fig. 4 (right).



**Fig. 5** Estimated intensity distribution for image 2. The input intensities have been filtered before using PR algorithm (left), intensity distribution estimation when filtering is performed during propagation (right).

The estimated intensity distribution, again for Image 2 is illustrated on Fig. 5. Here however, the input intensities have been filtered before using PR algorithm (left). Intensity distribution estimation when filtering is performed during propagation is plotted on Fig. 5 (right). Filtering removes details from the image.



**Fig. 6** Correlation coefficient of input phase vs. cut-off frequency of filter during propagation. Reference: Image 2.

Fig. 6 demonstrates the behavior of the correlation coefficient between the measured and the iterated intensity distribution depending on the cut-off frequency of the low-pass propagator filter and the initial phase. Here a uniform phase (Flat), a random phase with an autocorrelation width of nearly 5.1 pixels (RP5) as well as a delta autocorrelated random phase (RP1, one pixel wide) are applied. While the Flat and RP5 phase converge reliably even when no filtering is performed during the SM, the RP1 requires a filtering operation to converge.

## 5 Summary

We have found from these preliminary experimental results that best results were obtained when high resolution intensity images were input to the PR algorithm. Importantly however, it was necessary to perform spatial filtering during the implementation of the SM. This approach appears to provide robustness to the choice of initial phase.

## References

- [1] J.R. Fienup: Phase retrieval algorithms a comparison. *Appl. Optics*, Vol. 21, No. 15, 2758-2769, 1982;
- [2] J.R. Fienup: Phase retrieval algorithms for a complicated optical system. *Appl. Optics*, Vol. 32, No. 10, 1737-1746, 1993;
- [3] R. Dorsch, A. Lohmann, S. Sinzinger - Fresnel ping-pong algorithm for two plane computer-generated hologram display. *Appl. Optics*, Vol. 33, No. 5, 869-875, 1994;
- [4] G. Pedrini, W. Osten, Y. Zhang: Wave-front reconstruction from a sequence of interferograms recorded at different planes. *Opt. Lett.* 30, 833-835, 2005;
- [5] P. F. Almoró, S. G. Hanson: Random phase plate for wavefront sensing via phase retrieval and volume speckle field. *Appl. Optics*, Vol. 47, No. 16, 2979-2987, 2008;
- [6] D. P. Kelly, B. M. Hennelly, W. T. Rhodes, J. T. Sheridan: Analytical and Numerical Analysis of Linear Optical Systems. *Opt. Eng.* 45, Iss. 8, 088201-12, 2006