Phase measurement of a spatially varying polarization distribution

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Optical polarization manipulating elements usually generate additional phase terms together with the desired polarization distribution. For a polarization which is locally linear, an interferometric setup is presented to simultaneously measure both the additional relative phase as well as the polarization distribution.

1 Introduction

The measurement of the polarization and phase distributions of a wave field is an often posed problem [1,2]. Usually, the polarization is determined by measuring the Stokes parameters, but without access to the phase. Phase shifting interferometry (PSI) may be used to determine the phase, but without considering the polarization as a measurand. In this work, we will propose an interferometric approach for a spatially resolved measurement of both phase and polarization, as long as the local polarization at any point is linear. This is achieved by introducing a new set of algorithms similar to the well established ones used in PSI [3].

2 Two-beam interference with spatially varying polarization

Using a two-beam setup, the object beam with above-mentioned spatially variant but locally linear polarization $\Omega(x,y)$ and arbitrary phase $\Phi(x,y)$ can be described by the Jones-vector [4]

$$\vec{J}_O = u_O(x,y) e^{i\Phi(x,y)} \begin{pmatrix} \cos\Omega(x,y) \\ \sin\Omega(x,y) \end{pmatrix}. \quad (1)$$

The reference beam (linear polarization with angle ω_i and homogeneous phase φ_i, i referring to the different phase steps employed) is described as

$$\vec{J}_R^{(i)} = u_R e^{i\varphi_i} \begin{pmatrix} \cos\omega_i \\ \sin\omega_i \end{pmatrix}. \quad (2)$$

The resulting interference pattern on a detector can be easily derived and is given by

$$H_i = I_0 \left[1 + V\cos(\Phi - \varphi_i)\cos(\Omega - \omega_i)\right]. \quad (3)$$

To our knowledge, a method to simultaneously measure a combination of both the polarization and the phase of an object has not been established up to this point. Even the mere measurement of the phase may be hampered by the polarization of the light field: In eq. 3, the polarization Ω in the object beam may take any value; if, by chance or by purpose, the polarization $\Omega(x,y)$ is orthogonal to the polarization in the reference arm at (x,y), a complete loss of contrast in the intensity pattern H_i follows (Fig. 1) [5].

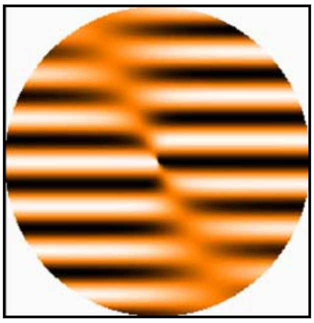

Fig. 1 *Simulated intensity distribution. The object wave is radially polarized. Due to orthogonal polarizations in object and reference wave, regions of low visibility and complete loss of contrast occur, and are a distinct feature in this image.*

To overcome this challenge, we propose to expand the principles of PSI by using additional polarization steps ω_i (i.e. different linear polarization angles in the reference beam) together with the common phase steps φ_i to determine phase and polarization simultaneously.

3 Sketch of the evaluation algorithm

First, eq. 3 is simplified and rewritten using

$$\vec{a}_i = \begin{pmatrix} 1 \\ \cos(\varphi_i + \omega_i) \\ \sin(\varphi_i + \omega_i) \\ \cos(\varphi_i - \omega_i) \\ \sin(\varphi_i - \omega_i) \end{pmatrix}; \vec{b} = \frac{I_0 V}{2} \begin{pmatrix} 2/V \\ \cos(\Phi + \Omega) \\ \sin(\Phi + \Omega) \\ \cos(\Phi - \Omega) \\ \sin(\Phi - \Omega) \end{pmatrix}, \quad (4)$$

yielding the expression

$$H_i = \vec{a}_i^T \cdot \vec{b}. \quad (5)$$

The entries *b(j)* (*j*=1…5), forming the vector ***b*** from eq. 4, contain the unknown values to be calculated to determine phase and polarization. Following eq. 4, it can be inferred that Φ and Ω are given by

$$\begin{aligned} \Phi &= \frac{1}{2}\left(\arctan\frac{b(3)}{b(2)} + \arctan\frac{b(5)}{b(4)}\right) \\ \Omega &= \frac{1}{2}\left(\arctan\frac{b(3)}{b(2)} - \arctan\frac{b(5)}{b(4)}\right) \end{aligned}. \quad (6)$$

To solve for ***b***, eq. 5 can be put into a different form:

$$\vec{c}_i = \vec{a}_i \cdot H_i = \left(\vec{a}_i \cdot \vec{a}_i^{\,T}\right) \cdot \vec{b} = \hat{A}_i \cdot \vec{b}\,. \quad (7)$$

Generally, it is not possible to invert the matrix $\boldsymbol{A}_i$. If, however, *N* images H_i are summed over with coefficients α_i, the equation can be solved for ***b*** by a suitable choice of the α_i, φ_i, and ω_i:

$$\begin{aligned} \sum_{i=1}^{N} \alpha_i \vec{c}_i &= \sum_{i=1}^{N} \alpha_i \hat{A}_i \cdot \vec{b} = \hat{A} \cdot \vec{b} \\ \hat{A}^{-1} \cdot \sum_{i=1}^{N} \alpha_i \vec{c}_i &= \left(\hat{A}^{-1} \cdot \hat{A}\right) \cdot \vec{b} = \vec{b} \end{aligned}. \quad (8)$$

The choice of the values *N*, α_i, φ_i, ω_i influences the robustness against positioning errors, non-linear behavior of the detector, and external vibrations. After a simplification, the entries of ***b*** can be written as

$$b(j) = \sum_{i=1}^{N} x_i^{(j)} \cdot H_i;\; j = 1 \ldots 5, \quad (9)$$

with $x_i^{(j)}$ being the weights of the respective images. Examples for different sets of $x_i^{(j)}$ – strongly depending on the choice of *N*, α_i, φ_i, ω_i – can be found in [6].

4 Setup

Now, φ_i and ω_i correspond to experimental settings of phase and polarization steps in the reference arm. While the setup used to generate the interferograms will be the topic of another paper, a short overview will be given in this proceeding. Fig. 2 shows the scheme of a Mach-Zehnder type setup enabling polarization and phase shifting interferometry.

A collimated laser beam forms the light source. As a beam splitter, a Wollaston prism is used. The prism was selected for its high quality polarization properties and therefore the possibility to adjust the visibility of the interferograms by controlling the intensity ratio between object and reference wave. In the reference arm, the linear polarization angle ω_i is being controlled by the rotation of a half-wave plate (HWP). Also, the mirror is moveable to allow for phase shifting. As a beam combiner, a specially designed phase grating is used. The influence of the grating parameters – such as the period, duty cycle, and etching depth – on TE and TM polarization have been investigated regarding diffraction efficiency and phase retardation, and the grating is designed to conserve any polarization patterns present in the incident beams.

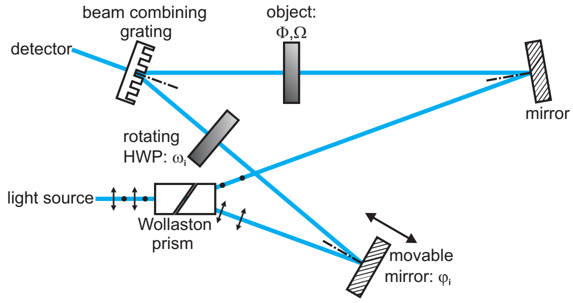

Fig. 2 *Scheme of a polarization and phase shifting two-beam interferometer adapted to the measurement task.*

To our knowledge, the main difference of this setup compared to previous measurement methods is the application of above-mentioned grating as a beam combiner, which minimizes the influence of the polarization distribution on the measurement. As a consequence, the setup deviates from the typical right angular geometries featuring small diffraction angles, leading to almost normal incidence on the mirrors, supressing the influence of the mirror reflections on the polarization states, as the Fresnel coefficients indicate. Using a Wollaston prism as a beam splitter instead of a second, identical grating is advantageous with respect to the adjustable visibility, but not necessary. Still, it provides the small angles needed for the setup geometry.

Literature

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