Tilt operator for harmonic fields and its application to propagation through plane interfaces

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This article focuses on the method to calculate the vectorial harmonic field on a tilted plane, which is referred as the tilt operator. Knowing the field distribution on one plane, one can find its distribution on any practically tilted plane by using tilt operator. In this article, the application of the tilt operator is presented in the case of the propagation of harmonic field through planar interface.

1 Introduction

Two rigorous operators will be introduced in this article: the tilt operator allows to calculate the field distribution on a tilted plane, which shares the origin with the original plane; the planar interface operator allows to calculate the reflected and transmitted field on both side of an interface with the knowledge of the incident harmonic field on the interface plane. By combining both operators, the propagation of a harmonic field through an arbitrary interface can be solved. Both operators are based on the idea of plane wave decomposition. In the following sections the fundamentals of them will be discussed.

2 Plane wave decomposition [1]

By Fourier transform, the field $V_l(\boldsymbol{\rho})$ in spatial domain and field $A_l(\boldsymbol{\kappa})$ in spatial frequency domain (k-domain) are related as

$$V_l(\boldsymbol{\rho}) = \mathscr{F}^{-1}\left[A_l(\boldsymbol{\kappa})\right] , \tag{1}$$

where $l = 1, 2, ..., 6$ represents the six components of an electromagnetic field. Eq. (1) describes the field on the $z = 0$ plane continuously. To obtain the full field everywhere in the current coordinate system in discretized and explicit form, we rewrite Eq. (1) in a vectorial form as

$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{M}\sum_{n=1}^{N}\frac{1}{2\pi}\mathbf{A}_{m,n}\mathrm{e}^{\mathrm{i}\mathbf{k}_{m,n}\cdot\mathbf{r}}\delta k_{x,m}\delta k_{y,n} , \tag{2}$$

with $\mathbf{A} = (A_1,\ A_2,\ A_3)^{\mathrm{T}}$. By examining Eq. (2), it can be found that only two vectors are needed to determinate a plane wave components. For example, plane wave component with specified index m and n can be determined by wave vector $\mathbf{k}_{m,n}$ and complex amplitude vector $\mathbf{A}_{m,n}$. Thus in this article, a vector pair $[\mathbf{k},\ \mathbf{A}]_{m,n}$ is usually used to specify a plane wave.

3 General rotations

A physical quantity, e.g., a vector, remains itself in different coordinate systems. In this section, we discuss the rotation of a Cartesian coordinate system. An arbitrary rotation of a Cartesian coordinate system can be decomposed into three basic rotations around the three axes. For example, after an arbitrary rotation the original vector $\mathbf{A}$ becomes $\bar{\mathbf{A}}$ as

$$\bar{\mathbf{A}} = \mathbf{T_z}\,\mathbf{T_x}\,\mathbf{T_y}\,\mathbf{A} , \tag{3}$$

where the three fundamental rotation are represented by $\mathbf{T_z}$, $\mathbf{T_x}$ and $\mathbf{T_y}$.

Next we apply the rotation on a plane wave component $[\mathbf{k},\ \mathbf{A}]_{m,n}$. Since a plane wave is fully-described by such vector pair, one only need to apply the same $\mathbf{T}$ on both $\mathbf{k}_{m,n}$ and $\mathbf{A}_{m,n}$ to obtain the new vector pair in the new coordinate system. The only thing that has to be ensured is that both coordinate systems share the same origin.

4 Tilt operator

In practice we are always dealing with a beam, which is composed by a set of plane wave components. On $x-o-y$ plane the beam can be sampled as $V_l^{\mathrm{original}}(\boldsymbol{\rho})$. By using fast Fourier transform (FFT) technique its vectorial spatial spectrum can be obtained as $A_{l;p,q}^{\mathrm{original}}$ on an equidistant sampling grid. The aim is to obtain the field distribution $V_l^{\mathrm{tilt}}(\bar{\boldsymbol{\rho}})$ on the tilted plane. To realize that we prefer to obtain the spatial spectrum $A_{l;m,n}^{\mathrm{tilt}}$ first. Here $A_{l;m,n}^{\mathrm{tilt}}$ is required to be sampled on an equidistant grid so that FFT can be applied.

Inspired by the idea of coordinate system rotation, it is easy to think of applying rotation operation on each plane wave components and compose them in the new coordinate system. The principle works but the nonlinearity [2] of a general rotational operation leads to a non-equidistant sampling grid finally, which disables the use of FFT. Thus in the following we propose a new method to avoid this.

Starting from the tilted coordinate system, the beam can be expressed as

$$\mathbf{E}^{\mathrm{tilt}}(\bar{\mathbf{r}}) = \sum_{m=1}^{M}\sum_{n=1}^{N}\frac{1}{2\pi}\mathbf{A}^{\mathrm{tilt}}_{m,n}\mathrm{e}^{\mathrm{i}\bar{\mathbf{k}}_{m,n}\cdot\bar{\mathbf{r}}}\Delta\bar{k}_{x,m}\Delta\bar{k}_{y,n}\,. \tag{4}$$

By setting $\bar{z} = 0$ one can see a similar form as in Eq. (1) and components of $\mathbf{A}^{\mathrm{tilt}}_{m,n}$ represent the spatial spectrums which are sampled on an equidistant grid. To obtain the spectrum, we first rotate the coordinate system to obtain the expression of each plane wave component in the initial coordinate system.

$$\mathbf{k}_{m,n} = \mathbf{T}^{-1}\,\bar{\mathbf{k}}_{m,n}, \quad \mathbf{A}^{\mathrm{rotate}}_{m,n} = \mathbf{T}^{-1}\,\mathbf{A}^{\mathrm{tilt}}_{m,n}\,. \tag{5}$$

These components can be assembled as

$$\mathbf{E}^{\mathrm{rotate}}(\mathbf{r}) = \sum_{m=1}^{M}\sum_{n=1}^{N}\frac{1}{2\pi}\Delta\bar{s}\mathbf{A}^{\mathrm{rotate}}_{m,n}\mathrm{e}^{\mathrm{i}\mathbf{k}_{m,n}\cdot\mathbf{r}}\,, \tag{6}$$

with $\bar{s} = \Delta\bar{k}_{x,m}\Delta\bar{k}_{y,n}$. By setting $z = 0$ Eq. (6) becomes

$$V^{\mathrm{rotate}}_{l}(\boldsymbol{\rho}) = \sum_{m=1}^{M}\sum_{n=1}^{N}\frac{1}{2\pi}\Delta\bar{s}A^{\mathrm{rotate}}_{l;m,n}\mathrm{e}^{\mathrm{i}\boldsymbol{\kappa}_{m,n}\cdot\boldsymbol{\rho}}\,. \tag{7}$$

Because of the nonlinearity of the rotation, $(k_{x,m},\ k_{y,n})$ are no longer distributed on an equidistant grid in general, while the initial spectrum $A^{\mathrm{original}}_{l;p,q}$ is given on an equidistant grid. To connect both fields an interpolation of the initial spectrum is necessary. By introducing several auxiliary points in the sampling grid, the corresponding sampling area around each sampling point can be determined and the initial field can be sampled as

$$V^{\mathrm{original}}_{l}(\boldsymbol{\rho}) = \sum_{m=1}^{M}\sum_{n=1}^{N}\frac{1}{2\pi}\Delta s_{m,n}A^{\mathrm{original}}_{l;m,n}\mathrm{e}^{\mathrm{i}\boldsymbol{\kappa}_{m,n}\cdot\boldsymbol{\rho}}\,. \tag{8}$$

The determination of the sampling area $\Delta s_{m,n}$ is shown in Fig. 1.

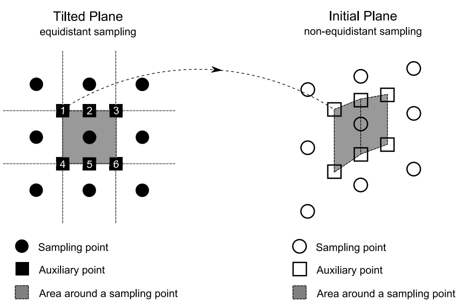

Fig. 1 *Effect of the nonlinear rotational operation* $\mathbf{T_x}$ *on the sampling points array in k-domain. From the tilted plane to the initial plane (from the solid circles to the hollow ones), it can be found that the equidistant sampling grid is changed into a non-equidistant one.*

Since Eq. (7) and (8) describe the same field distribution in one coordinate system, they should be identical. Thus the relation below can be found

$$A^{\mathrm{rotate}}_{l;m,n} = S_{m,n}\,A^{\mathrm{original}}_{l;m,n}\,, \tag{9}$$

where $S_{m,n} = \Delta s_{m,n}/\Delta\bar{s}$. The non-equidistantly sampled spectrum $A^{\mathrm{original}}_{l;m,n}$ can be obtained by an interpolation of $A^{\mathrm{original}}_{l;p,q}$, which is sampled on an equidistant grid. After that $A^{\mathrm{rotate}}_{l;m,n}$, from which we can easily obtain $A^{\mathrm{tilt}}_{l;m,n}$ by an inverse operation of Eq. (5). Finally the field on the tilted plane can be obtained by an inverse Fourier transform of $A^{\mathrm{tilt}}_{l;m,n}$.

5 Planar interface operator

The planar interface operator is based on Fresnel's equations. A general beam is first decomposed by a set of plane wave components. For each of them, a proper coordinate system can be found where Fresnel's equations can be applied.

Let's assume the incident field is known as a summation of plane waves $[\mathbf{k}^{\mathrm{I}},\ \mathbf{A}^{\mathrm{I}}]_{m,n}$. For each plane wave component, the first step is to find a certain $\mathbf{T}_{\mathbf{z};m,n}$ with a proper rotational angle $\alpha_{m,n}$, according to the wave vector $\mathbf{k}^{\mathrm{I}}_{m,n}$. Applying of $\mathbf{T}_{\mathbf{z};m,n}$ gives the wave vector and complex amplitude $[\mathbf{T_z}\mathbf{k}^{\mathrm{I}},\ \mathbf{T_z}\mathbf{A}^{\mathrm{I}}]_{m,n}$ of the plane wave in its convenient coordinate system. There the Fresnel equations are feasible. Then the reflected and transmitted field can be expressed as

$$\text{reflected}\ \left[\mathcal{R}_{\mathbf{k}}\left\{\mathbf{T_z}\mathbf{k}^{\mathrm{I}}\right\},\ \mathcal{R}_{\mathbf{A}}\left\{\mathbf{T_z}\mathbf{A}^{\mathrm{I}}\right\}\right]_{m,n}\,, \tag{10}$$

$$\text{transmitted}\ \left[\mathcal{T}_{\mathbf{k}}\left\{\mathbf{T_z}\mathbf{k}^{\mathrm{I}}\right\},\ \mathcal{T}_{\mathbf{A}}\left\{\mathbf{T_z}\mathbf{A}^{\mathrm{I}}\right\}\right]_{m,n}\,. \tag{11}$$

Then a $\mathbf{T}^{-1}_{\mathbf{z};m,n}$ rotation can be used to convert them back into the initial coordinate system.

6 Summary

Several simulations have been done with the optics software VirtualLab$^{\mathrm{TM}}$[3]. For example, the transmission and reflection at various incident angles, total internal reflection and Brewster effect. For more details please refer to [4].

References

[1] F. Wyrowski and M. Kuhn, "Introduction to field tracing," Journal of Modern Optics **58**(5-6), 449–466 (2011).

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[4] S. Zhang, D. Asoubar, F. Wyrowski, and M. Kuhn, "Efficient and rigorous propagation of harmonic fields through plane interfaces," Proc. SPIE 8429 (2012).