Geometrical optical design in phase space

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The phase space concept is a powerful method to evaluate the performance of an optical system. Since it is possible to observe the ray position and angle simultaneously. Especially in illumination design problems, where the transport of radiance is relevant, the phase space concept provides an interesting approach. Widely used geometries in illumination design are integrator rods and optical arrays. From the phase space analysis it is possible to check whether the target plane of the optical system is illuminated homogeneously.

1 Introduction

The phase space concept provides an entire picture of an illumination design. Therefore, it is an alternative tool, compared to ray-tracing, which can be used to evaluate optical designs within the geometrical optical limit. Furthermore it offers a different perspective on such systems. Integrator rods and optical arrays are frequently used components to homogenize radiation fields. Therefore, it is of interest to develop alternative methods to evaluate the performance of such systems besides ray tracing.

2 Phase space

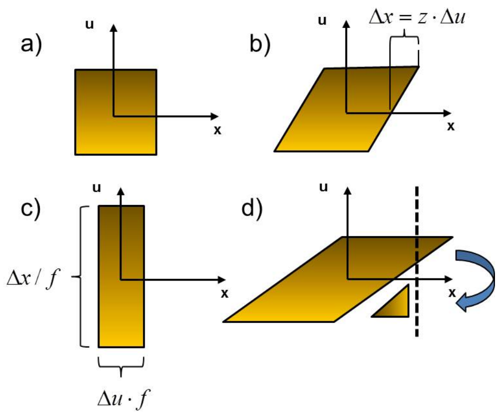

Fig. 1 *a) Initial distribution in phase space. b) Distribution after free propagation. c) Distribution after propagation through a lens. d) Back-folded distribution due to the reflection on a mirror surface.*

In the general optical limit, the phase space represents the distribution of rays in position and angle within an optical system [1], [2].
The energy within a certain phase space volume is the radiance L(x,u). From the projection of the radiance to the spatial and angular axis, the irradiance E(x) and the intensity I(u) can be calculated directly. Generally, the radiance is a 4-dimensional quantity but within the context of this paper we will restrict ourselves to two dimensions. The radiance is conserved, thus an optical system with only reflective and refractive elements can only redistribute the phase space volume. As we will see, geometrical optical systems can merely lead to a deformation in phase space.
In phase space a single ray is represented as point in the phase space diagram. If we employ the paraxial ABCD matrix formalism or the free space propagation [3], it is now possible to illustrate transformations according to different optical systems in order to understand more complex systems such as integrator-rods and optical arrays. A free propagation of rays corresponds to a shear of the initial distribution in phase space according to the paraxial propagation laws.

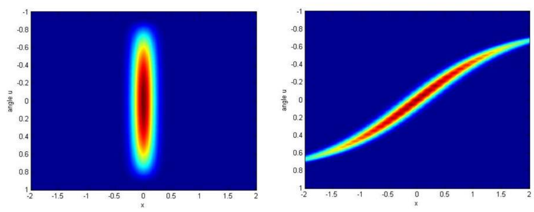

Fig. 2 *Free propagation in phase space. In the left picture, the initial Gaussian angular distribution is shown. The right diagram illustrates the distribution after 1 mm of free propagation.*

If rays propagate through a lens, the corresponding phase space distribution is rotated by 90 degrees as a result of the Fourier transformation performed by a lens [1]. In presence of a mirror the rays are reflected on the mirror surface which leads to a back-folded distribution in phase space.
In the case of non-paraxial free propagation the phase space distribution is additionally distorted due to the nonlinear behavior of the ray in angle. A general light distribution has to be propagated by using the complex Wigner distribution [4]. With the basic principles explained above it is now possible to understand also more complex systems.

3 Integrator-rods

Integrator-rods are widely used in illumination designs due to their light mixing properties. Many different geometries are known, such as conic or circular rods. Within the context of this proceeding we will discuss only simple symmetric rectangular rods.
If we consider a point source with a Gaussian angular distribution at the entrance of the rod the incident light will be reflected multiple times at the inner side walls of the rod. With each reflection the phase space distribution is back folded. The free propagation between two reflections leads to an additional shear of the phase space volume. From the projection to the spatial axis the irradiance can be calculated directly. The homogenization effect after the action of the rod can be seen in a homogeneous irradiance on the target plane [5].
Thus the distribution in phase space at the exit of the rod is uniform in position and segmented in angle. Therefore, by looking into the rod from the exit surface, multiple reflections can be seen, the well known "‘kaleidoscope effect"’ which creates multiple patterns of the input distribution.

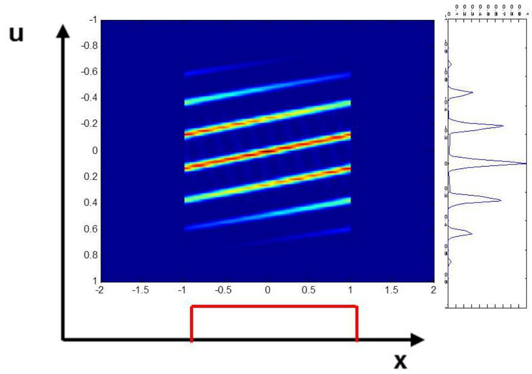

Fig. 3 The diagram exhibits the phase space distribution of an integrator-rod at the exit plane of the rod after multiple reflections. The red curve shows the projection to the spatial axis, the irradiance, which is constant. The right diagram illustrates the distribution at one point of the exit surface of the angular intensity.

4 Optical arrays

Optical arrays separate the incoming light distribution into channels according to the aperture width of the micro lenses [6]. If we consider one single channel of an optical array, the incoming light is segmented and refracted by the different lenses. Each phase space segment is rotated due to the action of the micro-lenses resulting in a structured line pattern in phase space. Due to the compensating influence of the second micro-lens array, no shearing can be observed in fig. 4 although we have a free propagation between the arrays. The final integrator lens introduces a rotation by 90 degrees according to a Fourier transformation. From the final phase space distribution it can be seen that in the target plane all channels are superimposed and the irradiance is uniform.

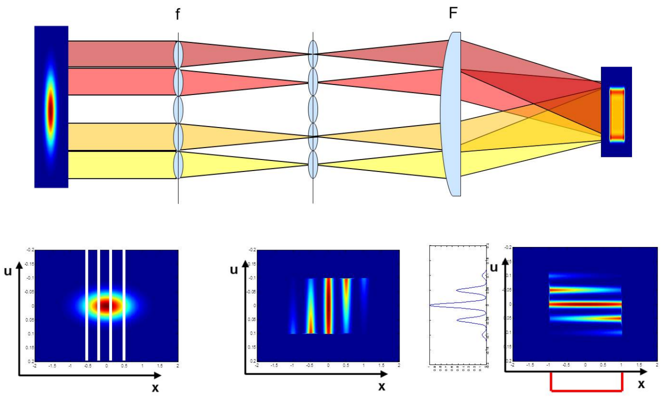

Fig. 4 In the figure, the optical design of an array illumination system, consisting of two optical arrays and an integrator lens, with the corresponding phase space pictures is shown.

5 Summary

The phase space approach provides a new perspective into illumination optics. The optical designer can use the phase space diagram to illustrate the basic principles of optical components in angle and position simultaneously. It is conceivable to apply this concept to more complex systems, like solar concentrators, in order to evaluate their performance.

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