

# Determining interpolation errors of angle encoders by error-separating shearing techniques

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When radial gratings are used in angle encoders, interpolation errors at small angular scales – due to the subdivision of the angular interval between adjacent grating lines into smaller intervals – are an important error source. We demonstrate the first ever adaptation and application of error-separating shearing techniques to the accurate determination of interpolation errors.

## 1 Introduction

Error-separating shearing techniques [1-2], by applying defined angular offsets between angle measuring systems, offer a unique opportunity to separate the errors of both systems and, therefore, to calibrate them without recourse to external standards. We demonstrate the first adaptation of this approach to the determination of the interpolation error of a radial grating. To put this work into perspective, we include a more general discussion of error sources of angle encoders and their calibration.

## 2 Primary angle standard WMT 220

At PTB, the WMT 220 angle comparator serves as the primary national standard for the realisation of the SI unit *radian* of the plane angle in Germany and is used for the most accurate calibrations of angle artefacts and measuring instruments [3].

The standard uncertainty of the calibration of the WMT 220 is of the order of  $u = 1$  milliarc (5 nrad) and has been verified by various internal comparisons (of cross- and self-calibration) and by comparisons with independent, external partners, which all demonstrate consistency at a level of  $< 1$  milliarc rms ( $< 5$  nrad rms) [4]. Autocollimators can be calibrated down to a standard uncertainty of  $u = 5$  milliarc (25 nrad). [5-7].

The WMT 220 consists of a precision air bearing rotary table equipped with a radial phase grating (400 mm in diameter) and a measuring system with eight interferential reading heads which are distributed at regular (45 degree) intervals. Additionally, eight auxiliary heads are arranged in diametrically opposed pairs to realise angle intervals of  $360 \text{ degrees} / 2^n$  with  $n=1-7$ , with the smallest being 2.81 degrees, see [3] for further details.

The grating consists of  $2^{17}$  graduation lines with angular separations between adjacent lines of 9.89 arcsec. Each reading head provides sinusoidal signals with twice the graduation frequency,

i.e.,  $2^{18}$  signals per  $360^\circ$ . These signals are further subdivided by the heads' electronics by a factor of  $2^{12}$  to obtain an effective resolution of approx. 1.2 milliarc per head ( $2^{30}$  intervals in  $360^\circ$ ).

As this interpolation is not error-free, a proprietary technology developed by the manufacturer, Johannes Heidenhain, Germany, is used to evaluate and correct the errors. Error components of low Fourier orders are determined directly from the data acquired by the heads during operation, while higher Fourier orders are corrected by the use of archived data derived from runs during which the grating rotates at a constant angular velocity.

With the WMT 220, the compensation of the interpolation errors is quite successful; residual errors of only 0.6 milliarc rms (3 nrad rms) remain after averaging the measurements by all eight primary heads. For other angle encoders, however, interpolation errors are often a major error source and novel approaches for determining them at low levels of uncertainty are urgently needed.

## 3 Error sources

The systematic measurement deviations of each reading head of the WMT 220 are caused by the following main sources (in order of effect size):

1. The graduation error of the radial phase grating due to manufacturing limits.
2. The eccentricity error due to the systematic misalignment between the centre of the grating and the mechanical rotation axis.
3. Residual lateral shifts of the grating when it is rotated under the fixed geometrical arrangement of the reading heads due to mechanical tolerances of the bearing.
4. Interpolation errors due to the subdivision of the angular interval between adjacent grating lines into smaller intervals by electronics.

- Non-uniformities in the response of the heads to the grating's graduation error due to manufacturing tolerances and limits in their geometrical alignment.

## 4 Novel solutions

### 4.1 Graduation error

For determining the graduation errors of the radial grating, two independent methods are available: the classical cross-calibration (against a built-in or external secondary angle encoder) and self-calibration. At PTB, extensive research on the fast and precise in situ self-calibration of angle encoders has been carried out [8-10]. This method relies on a suitable geometric arrangement of multiple reading heads which read out the radial grating of the angle encoder at different angular positions.

Self-calibration offers a number of advantages, foremost that it is independent of auxiliary devices (such as, e.g., a secondary angle encoder or a polygon). As part of the family of circle division methods, it utilises the full circle as a natural, error-free angular standard which provides independence from external reference standards. Furthermore, self-calibration is fast and therefore our preferred calibration method.

### 4.2 Eccentricity error

The self-calibration method described in Section 4.1 also determines the grating's eccentricity error.

### 4.3 Lateral shift

We have developed an improved self-calibration analysis which accounts for lateral shifts of the grating during its rotation for the first time [10]. The new analysis eliminates lateral shifts during self-calibration to improve the determination of the grating's graduation errors. During operation of the WMT 220, the influence of the lateral shifts is eliminated due to averaging measurements provided by diametrically opposed pairs of reading heads.

### 4.4 Interpolation error

We are currently investigating the adaptation of advanced error-separating shearing techniques [1-2] to the calibration of angle measuring devices, e.g., the WMT 220 primary angle standard and a precision electronic autocollimator. Shearing techniques, by applying defined angular offsets between both systems, offer a unique opportunity to separate their errors and, therefore, to calibrate both without recourse to any external standard.

First experimental tests show very promising results. Error separation has been achieved at a level of approx. 1 milliarc (5 nrad) with room for further improvement. Figure 1 shows an example of the residual interpolation error of the WMT 220 after averaging the signals by the eight primary reading heads (as is done during its operation).

The curves show the interpolation errors with the correction of the errors described in Section 2 switched on (blue) and off (black). The rms values are of the order of 1 milliarc rms (5 nrad rms) and 10 milliarc rms (50 nrad rms), respectively.

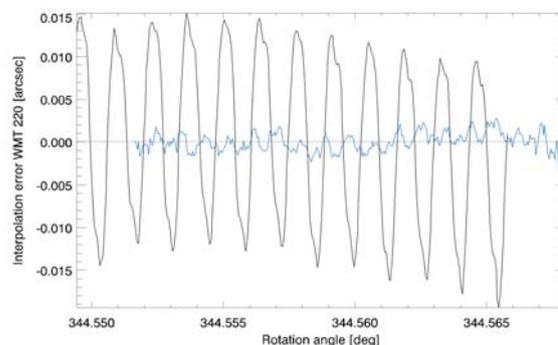


Figure 1: Residual interpolation error of the WMT 220 with error correction on (blue) / off (black).

### 4.5 Non-uniformity

After accounting for the influences of all known error sources, residual deviations in the measurements of the reading heads of the WMT 220 at a level of 1 milliarc rms (5 nrad rms) have been detected. Due to their small magnitude, it is entirely plausible that they can be attributed to residual interpolation errors and that non-uniformities in the heads' responses are considerably below that level.

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