Discrete numerical implementation of the Fresnel transform in Cartesian and cylindrical coordinate systems

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The Fresnel transform is a diffraction integral used in optics to calculate the propagation of a wave field in the paraxial domain. It is possible to express the Fresnel transform in Cartesian or cylindrical coordinates. Often the choice of coordinate system depends on the optical problem being examined. In this proceeding we examine the often overlooked problem of representing an optical field discretely using cylindrical coordinates.

1 Introduction

The Fresnel transform (the Fresnel diffraction integral) is often used in the optics, to calculate the propagation of wave in the near field. Many numerical methods are used to do this transform, like direct method, spectral method etc. In this proceeding we contrast the discrete numerical implementation of the Fresnel transform in two different coordinate and analyze the replica properties of the two solutions.

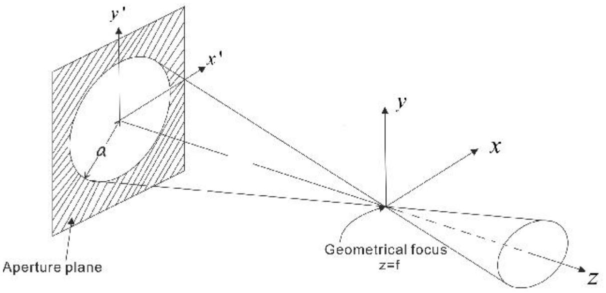

Fig.1. *Schematic representation of the diffraction geometry*

2 Mathematical description of the Fresnel transforms

Using Fresnel transform we can calculate the light propagation from one plane (x', y') to another (x, y), as shown in Fig. 1. We have the Fresnel transform in Cartesian coordinate with,

$$A(x,y,z)=\frac{e^{ikz}}{i\lambda z}\iint_S A_0(x',y')e^{\frac{ik}{2z}\left[(x'-x)^2+(y'-y)^2\right]}dx'dy' \qquad (1)$$

Which λ is the wave length, and z is the distance between the two planes, k is the wave number. Due in part to the rectangular coordinate systems associated with CCD arrays, the Cartesian system is widely used to represent a digital signal. However in other spherical aperture case, like a lens system, the cylindrical coordinate is more suitable to be chosen. The Fresnel transform in cylindrical coordinate is written as [1],

$$A(r,\phi,z)=\frac{e^{ik(z+\frac{r^2}{2z})}}{i\lambda z}\iint_S A_0(R,\theta)e^{\frac{ik}{2z}\left[R^2-2Rr\cos(\theta-\phi)\right]}RdRd\theta \qquad (2)$$

Usually these integrals have no analytical solutions, and they need to be solved numerically. Here we use discrete implementation to rewrite these equations with,

$$A(x,y,z)=\frac{e^{ikz}}{i\lambda z}\sum_{n=1}^{N}\sum_{m=1}^{M}A_0(Xn',Yn')e^{\frac{ik}{2z}\left[(Xn'-x)^2+(Yn'-y)^2\right]}\Delta x'\Delta y'$$

(3)

$$A(r,\phi,z)=\frac{e^{ik(z+\frac{r^2}{2z})}}{i\lambda z}\sum_{n=1}^{N}\sum_{m=1}^{M}A_0(R_n,\theta_m)e^{\frac{ik}{2z}\left[R_n^2-2R_nr\cos(\theta_m-\phi)\right]}R_n\Delta R\Delta\theta$$

(4)

The sampling type in Eq. (3) is $\Delta x'=2a/N$, $\Delta y'=2a/M$, and in Eq. (4) is $\Delta R=a/N$, $\Delta\theta=2\pi/M$, *N* and *M* are the number of samplings. With Eq. (3) or (4) we have discrete implementation of the Fresnel transform in two coordinate.

3 Simulation

In this section, we calculate the propagation of a converging wave with the focus *f*. So we have the input plane in Cartesian coordinate,

$$A_0(x',y')=e^{\frac{-ik}{2f}(x'^2+y'^2)},$$

(5)

or in cylindrical coordinate,

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$$A_0(R,\theta) = e^{\frac{-ik}{2f}R^2}. \qquad (6)$$

Substituting Eq. (5) into Eq. (3) and Eq. (6) into Eq. (4) with *N*=50, *M*=50. We get the results as shown in Fig. 3 and Fig. 4. For both results, we have infinite replicas beside the original solution, however the replicas are significant different in the two cases. In the Cartesian coordinate, we have the numerical solution A_N [2],

$$A_N(x,y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} A\left(x - \frac{n\lambda z}{\Delta x'}, y - \frac{m\lambda z}{\Delta y'} \right). \quad (7)$$

We see that replicas are separated by $\lambda z / \Delta x$ in *x* axis, and $\lambda z / \Delta y$ in *y* axis. The replicas are identical as the original one (located at *x*=0, *y*=0).

In the cylindrical coordinate, the numerical solution is written as [3],

$$A_N(r,\phi,z) = A(r,\phi,z) + \frac{1}{2\pi}\Big(\sum_{n=-\infty}^{-1} + \sum_{n=1}^{\infty} \Big)* \\ A(r,\phi,z) ** \frac{i2\pi \frac{n}{\Delta R}}{\left[(2\pi r/\lambda z)^2 - (2\pi n / \Delta R)^2 \right]^{\frac{3}{2}}} \quad , (8)$$

which ** is the convolution operation. In this case, the replicas are distributed circinally, around the original one, they located at $r = n\lambda z / \Delta R$, *n*=1, 2, 3… These replicas are not as same as the original one, and there power are twice the original one, because + and – order replicas overlap themselves at the same location

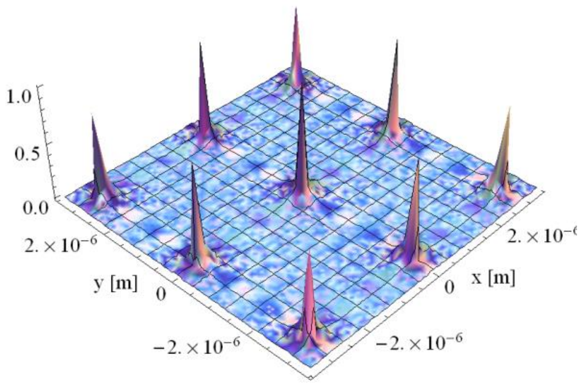

***Fig.2**. Intensity distribution at the focus plane using the Fresnel transform in Cartesian coordinate.*

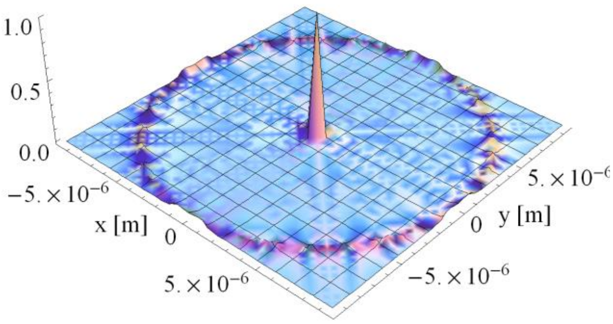

***Fig.3**. Intensity distribution at the focus plane using the Fresnel transform in Cartesian coordinate.*

4 Conclusion

Here we present two different kinds of discrete numerical methods to calculate the Fresnel diffraction integral, in Cartesian coordinate and in cylindrical coordinate. Both methods give the replicas at the output plane. However the replicas are significant different, every order of replicas in Cartesian coordinate is just like the origin, but in the cylindrical coordinate system, the replicas with the same order locate at the same place and are built circinally. In practice it must be ensure that enough sampling to be taken to separate the replicas and get the accurate results

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