Iterative phase retrieval: a numerical investigation

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Estimating the complex amplitude of a coherent optical wavefield is important in many metrology applications. One way to recover the phase information from measured intensity distributions are phase retrieval (PR) algorithms. Here we develop a numerical technique that accurately simulates the propagation of speckle fields and investigate the convergence properties of a PR algorithm.

1 Introduction

Determining the complex amplitude of a coherent monochromatic optical wavefield is necessary e.g. in many biological imaging and speckle metrology applications. Although digital holography can provide a solution, a coherent reference wave is required with additional optics and hence it is of interest to pursue other techniques that rely solely on intensity measurements. In this study we examine iterative Phase Retrieval (PR) techniques [2-4]. These PR techniques can converge to the correct phase distribution, however the convergence is dependent on a variety of experimental conditions such as the initial guess at the phase distribution or the number of intensity distributions used in the PR algorithm. For biological applications a constant phase often serves well as an initial phase estimate. However this approach will fail when we attempt to estimate the phase distribution of a speckle field.

We wish to determine some of the convergence properties of the PR algorithm when we capture a series of speckle fields at different axial planes, as depicted in Fig. 1. So as to avoid any experimental errors, such as CCD shot noise or misalignment of optical elements, we pursue a purely numerical investigation. We first examine how we can correctly propagate speckle fields numerically. Having established a robust numerical scheme we then turn our attention to modeling the experiment shown in Fig. 1, and examine the convergence of the PR algorithm for a range of different realistic experimental conditions.

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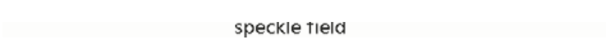

Fig. 1 *Experimental setup for evaluation of speckle field propagation and phase retrieval.*

2 Numerical propagation of speckle fields

Care must be taken when simulating the propagation of speckle fields numerically. From Ref. [1], the following relation can be used as a rule of thumb to determine the output extent (SE) of input field in diffraction plane:

$$SE = 2(\lambda z f_{\max} + \alpha_z) \tag{1}$$

Let us now examine how we may apply that relation to the random field, $U(X)$, in the input plane of Fig. 1. The parameter f_{max} in Eq. (1) is the highest spatial frequency component of $U(X)$ and α_z is related to the extent of the aperture, z to the propagation distance and λ stands for the wavelength of the monochromatic coherent light source. If this input field is given a different random phase value at each pixel, then f_{max} results as: $f_{max} = 1/2\delta X$. Using the spectral method to propagate the field we can guarantee that an accurate diffraction pattern is calculated provided that the numerical extent of the field is greater than SE. This can be ensured using zero-padding. With this numerical approach we can reproduce the speckle statistics derived from analytical considerations [1]. Some numerical results are presented below.

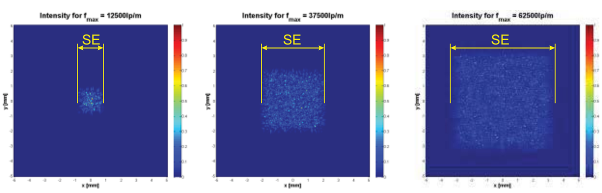

Fig. 2 *Influence of f_{max} on SE; z = 100mm, δX = 8μm, N = 1264²: f_{max} = 12.5 (left), 37.5 (center) as well as 62.5 (right) lp/mm. The calculated value of SE matches the output extent (width/length) of numerically calculated speckle field.*

the propagated speckle field. The parameter, f_{max}, can be further controlled by a low pass filtering operation in the spatial frequency domain.

Numerical calculation of diffraction fields produces an infinite set of replicas in the output plane. When using the spectral method (SM), these replicas are separated spatially from each other by $1/\delta v$, where δv is the spacing between adjacent samples in the spatial frequency domain. Zero padding makes δv smaller and hence increases the separation between output replicas. Once $1/\delta v \geq SE$ we have accurate results; see Fig. 3.

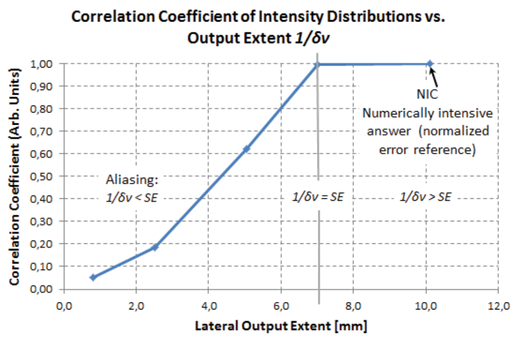

Fig. 3 *Numerically intensive answer (NIC) is a numerical result with significant zero padding to ensure no aliasing. NIC is compared with other calculations for a range of zero padding to determine an error ratio for* $1/\delta v < SE$.

We are now in a position to examine the iterative PR algorithm for a range of different experimental conditions and in an “ideal” noise-free environment.

3 Phase retrieval

Consider the setup shown in Fig. 1. A series of intensities at different optical planes are captured and used as input to the algorithm [5].

We now examine how the PR algorithm converges under different experimental conditions: (1) The effect of reducing the spatial frequency extent at the input plane, (2) Starting with the correct initial phase, phase noise is added until the solution no longer converges to the global (i.e. the correct phase distribution) but a local minimum, (3) The effect of more intensity captures, (4) The distance between the axial captures, and finally (5) The independence on the further lateral expansion of the retrieval planes by zero padding above the SE demands of Eq. 1.

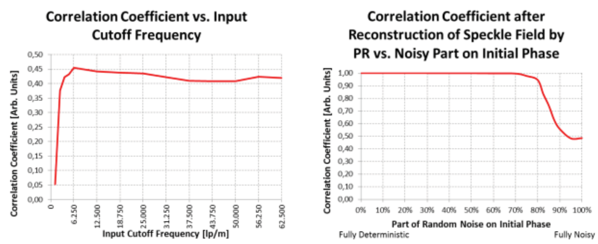

Fig. 4 *Influence of the spectral input bandwidth (left) and noise on initial phase (right).*

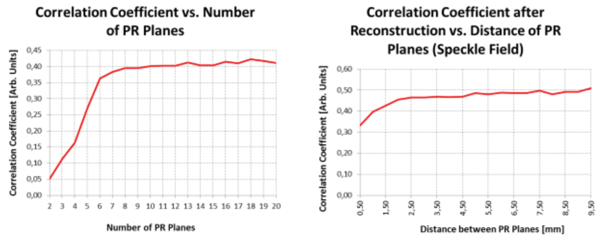

Fig. 5 *Influence of number of PR planes (left) and of axial distance between PR planes (right).*

4 Summary

Our results indicate that it is very difficult to converge to the correct phase distribution for the experimental setup shown in Fig. 1. As we increase the number of capture planes, the correlation coefficient (between the correct phase and that predicted by the PR algorithm) does improve but appears to increase asymptotically towards unity. A similar trend occurs when we keep the number of capture planes constant but increase the uniform axial distance between them. When we start with the correct initial phase plus a small random phase noise, the PR algorithm converges to the correct answer. As the magnitude of the random phase noise is increased, the PR algorithm still converges up to a threshold point.

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