Smooth field decomposition and its application to scattering phenomena

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In this work, the authors partially illustrate the potential of a smooth field decomposition as a simulation tool in optics through two scattering examples: in the first example the decomposition is applied in the space domain; in the second, in the frequency domain.

1 Theoretical foundation

The basic foundation on which this method is sustained is the framework provided by field tracing: light represented in the form of electromagnetic fields throughout the optical system, accessible at any given x, y plane (assuming z to be the direction of propagation). We refer those readers who wish to know more about the field-tracing concept to the work of Wyrowski *et al.* [1].

The aim of the smooth decomposition is to divide a field given in an x, y plane of the system, ($V_\ell(x,y) = E_x, E_y, E_z, H_x, H_y, H_z$, with $\ell = 1, \ldots, 6$) into **independent** subfields $V_{\ell,(i,j)}(x,y)$. Please note that, although the formulas are henceforth given in the space domain, their equivalent in the frequency domain is straight-forward to express, with just a substitution of k_x, k_y for x, y. Mathematically, the decomposition can be described as follows:

$$V_\ell(x,y) = \sum_{i,j} V_{\ell,(i,j)}(x,y) = \sum_{i,j} \Omega_{i,j}(x,y)\ V_\ell(x,y) \tag{1}$$

where $\Omega_{i,j}(x,y)$ represents a set of basis functions, i.e., the mathematical agents of the decomposition. It can be mathematically proved that the basis functions must fulfil the following condition so that all the information contained within $V_\ell(x,y)$ is preserved:

$$\sum_{i,j} \Omega_{i,j}(x,y) = 1 \quad \forall (x,y) \tag{2}$$

Finally, there is just the question of what function form we choose for these basis functions. The most straight-forward option would be to divide the x, y plane into perfectly delimited sectors, so that each of the basis functions is equal to 1 in their allotted sector and 0 everywhere else. If there is just one basis function per sector, and the basis functions cover the whole area of definition of the field, condition 2 is evidently upheld. The consequent mathematically sharp edges, however, make this a less-than-ideal choice: the infinitesimal sampling distance needed to correctly sample them is impossible to achieve on a computer. If bent on using these rect-type basis functions, the user will encounter a series of nasty numerical effects in their simulations, which stem from this very sampling issue.

We propose to overcome these shortcomings through the introduction of an overlapping cosine edge into our basis functions. This is shown schematically in 1D in Fig. 1. For the full mathematical expression of the basis functions, we refer the reader to the work of Asoubar *et al.*, [2].

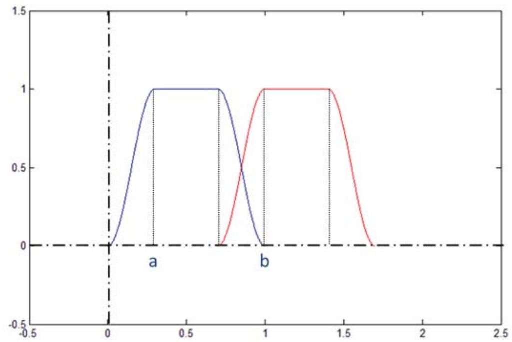

Fig. 1 *The figure shows a 1D schematic of the basis functions used in the decomposition. Arbitrary units are used.*

2 Numerical examples

2.1 Space domain: diffracting stop

A large plane wave encounters a small, sharp-edged stop which will diffract the wave. We seek to know the resulting field at a target plane some distance away from the stop.

The sharp edges of the stop mean that its sampling distance will be smaller than the sampling distance of the input field. Therefore, the input field will have to be resampled upon meeting the stop and, because the field is assumed to be large in extension, the total number of sampling points of the field will

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suffer a considerable increase. This obviously entails an increase of the numerical effort of the simulation, as the field then needs to be propagated to the target plane.

We propose to reduce the numerical effort of the simulation, and to thus improve the speed, by decomposing the input field just before it is resampled at the stop, into two subfields: one inner part, which will encounter the stop and be resampled, and one outer part, which will continue **with the original sampling** to the target plane without ever meeting the stop. The resulting much lower total number of sampling points will decrease computation effort and therefore increase speed.

Fig. 2 shows a simulation example which illustrates the above. The results of both approaches are given, as well as the data of the total numerical effort for comparison purposes.

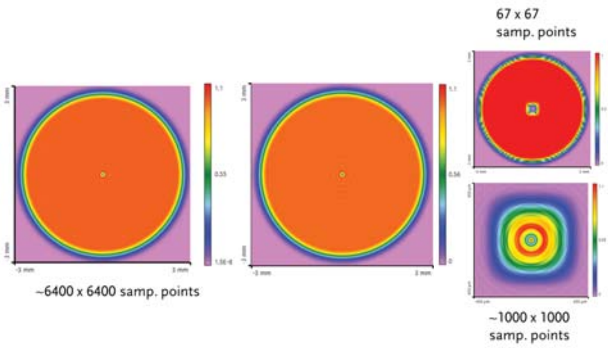

Fig. 2 *The figure shows the simulation results of a large plane wave interacting with a small stop. On the left without decomposition (reference), on the right the inner and outer subfields, and in the centre the addition of both subfields in order to be able to compare the result with the reference case. The reference case needs approx. 40960000 sampling points, while the decomposition needs just 1004489. All results are shown at the target plane. The magnitude represented is amplitude of E_x (in arbitrary units). Simulations carried out with VirtualLab [3].*

2.2 Frequency domain: scattered field through lens

In this example we assume a scattered field propagating through a real spherical lens and into the focus. Because of the dimensions of the lens, the propagation of the field through it must (realistically) be done using geometric-based propagation (geometric field tracing). However, the input field considered in this example does not fulfil the conditions of geometric field tracing, due to its scattered nature. The alternative would be to perform a fully fledged plane-wave decomposition of the input field for a rigorous result.

We propose a middle-ground situation: by decomposing the scattered input field using our method in the frequency domain into $N \times M$ subfields, we generate a set of *parabasal subfields* [2], i.e., fields whose spectrum deviates from that of a perfect plane wave by a certain Δk assumed to be small enough for the hypotheses of geometric field tracing to be (approximately) fulfilled. By adjusting the integers N and M (i.e., the number of subfields of our decomposition) we obtain a full spectrum of options between the completely erroneous result (no decomposition) and the rigorous one (plane-wave decomposition), which allows the user to fine-tune the accuracy and speed of their simulation according to their particular requirements.

As a first test for this method, we use as input field one scattered by a periodic scatterer. In this particular case, a 5x5 decomposition in the frequency domain will be (practically) equivalent to a plane wave decomposition, if each subfield contains just one of the spots of the frequency spectrum. Fig. 3 shows the difference in the result when the decomposition is applied, and when no decomposition at all is used (a case which we *a priori* know is wrong, as explained above).

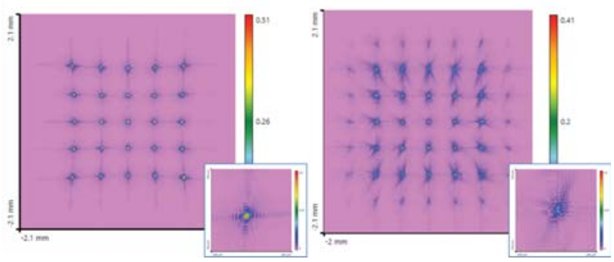

Fig. 3 *The figure shows the amplitude of E_x (in a.u.) at the focus of the lens when: left –the decomposition has been applied; right –no decomposition is used, and the scattered field is (erroneously) propagated through the lens as-is. Note the differences in the results. Simulations carried out with VirtualLab [3].*

3 Acknowledgements

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