Comparison of Modelling Techniques for Multimode Fibers and Its Application to VCSEL Source Coupling

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We present a geometrical optics based approach to model multimode fiber. This approach, which is mathmatically based on Runge-Kutta methods, handles light field information, e.g. amplitude and polarization, and works effeciently. Finally we model a VCSEL source coupled into a multimode fiber by using the well known split-step method and our approach. Comparison of both approaches is provided.

1 Introduction

Ray tracing and split-step method [1] are the most efficient techniques to model multimode fiber. Both concepts of the ray tracing and the split-step method are widely used in the modeling of multimode fibers. Both techniques work under specific assumptions: (1) Ray tracing allows modeling of non-paraxial situations but ignores diffraction dominated propagation effects as well does not handle light field information like polarization. (2) The split-step method assumes a paraxial propagation approach and by that ignores polarization crosstalk.

In this work, we propose a generalization of the ray tracing approach to an efficient geometrical optics field tracing technique, which enables the inclusion of polarization effects including crosstalk within the fiber. We discuss and compare the different techniques theoretically and at the example of the coupling of a VCSEL source into a multimode fiber (OM3, OM4). We calculate the coupling efficiency with respect to adjustment tolerances.

2 Geometric field tracing in fiber

The ray tracing equation for graded-index media [2] is

$$\frac{d}{ds}[\tilde{n}(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}] = \boldsymbol{\nabla}\tilde{n}(\boldsymbol{r}) \tag{1}$$

where $n(\boldsymbol{r})$ is the refractive index and s denotes the arc length of ray.

The field tracing equation for graded-index media [2] is

$$\tilde{n}(\boldsymbol{r})\frac{d\tilde{\boldsymbol{u}}(\boldsymbol{r})}{ds} = -[\tilde{\boldsymbol{u}}(\boldsymbol{r})\cdot\frac{\boldsymbol{\nabla}\tilde{n}(\boldsymbol{r})}{\tilde{n}(\boldsymbol{r})}]\tilde{n}(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds} \tag{2}$$

where $\tilde{\boldsymbol{u}}(\boldsymbol{r})$ denotes the normalized electric field at position $\boldsymbol{r}$:$\tilde{\boldsymbol{u}}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}(\boldsymbol{r})/||\tilde{\boldsymbol{E}}(\boldsymbol{r})||$.

Our approach solves the Eq. (1) and (2) simultaneously by using Runga-Kutta methods(RK) [3]. To simplify the equation, we define two assistant parameters: (i) $\boldsymbol{D}(\boldsymbol{r}) = \boldsymbol{\nabla}\tilde{n}(\boldsymbol{r})$; (ii)$\boldsymbol{T}(\boldsymbol{r}) = \tilde{n}(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}$.

Starting from the known point ($\boldsymbol{r}_0$, $\boldsymbol{T}_0$, OPL_0), we can calculate ($\boldsymbol{r}_i$, $\boldsymbol{T}_i$, OPL_i) by using RK which can be formulated as [4]

$$\begin{cases} \boldsymbol{r}'_i &= \boldsymbol{r}_i + \frac{\boldsymbol{T}(\boldsymbol{r}_i)}{\tilde{n}(\boldsymbol{r}_i)}\frac{\Delta s}{4} + \\ & \quad \frac{\Delta s^2}{32}\left\{\frac{\boldsymbol{D}(\boldsymbol{r}_i)}{\tilde{n}(\boldsymbol{r}_i)} - \frac{\boldsymbol{T}(\boldsymbol{r}_i)[\boldsymbol{D}(\boldsymbol{r}_i)\cdot\boldsymbol{T}(\boldsymbol{r}_i)]}{\tilde{n}^3(\boldsymbol{r}_i)}\right\} \\ \boldsymbol{r}_{i+1} &= \boldsymbol{r}_i + \frac{\boldsymbol{T}(\boldsymbol{r}_i)}{\tilde{n}(\boldsymbol{r}_i)}\frac{\Delta s}{2} + \\ & \quad \frac{\Delta s^2}{8}\left\{\frac{\boldsymbol{D}(\boldsymbol{r}_i)}{\tilde{n}(\boldsymbol{r}_i)} - \frac{\boldsymbol{T}(\boldsymbol{r}_i)[\boldsymbol{D}(\boldsymbol{r}_i)\cdot\boldsymbol{T}(\boldsymbol{r}_i)]}{\tilde{n}^3(\boldsymbol{r}_i)}\right\} \\ \boldsymbol{T}(\boldsymbol{r}_{i+1}) &= \boldsymbol{T}(\boldsymbol{r}_i) + \frac{\Delta s}{12}(\boldsymbol{D}(\boldsymbol{r}_i) + 4\boldsymbol{D}(\boldsymbol{r}'_i) + \boldsymbol{D}(\boldsymbol{r}_{i+1})) \\ OPL(\boldsymbol{r}_{i+1}) &= OPL(\boldsymbol{r}_i) + \frac{\Delta s}{8}(\tilde{n}(\boldsymbol{r}_i) + 2\tilde{n}(\boldsymbol{r}'_i) + \tilde{n}(\boldsymbol{r}_{i+1})) \end{cases} \tag{3}$$

where $\boldsymbol{r}_i$ is the position at discretized calculation grid and Δs is the incremental value of arc length s. To synchronize the calculation of both $\boldsymbol{r}$ and $\tilde{\boldsymbol{E}}$, the arc length between $\boldsymbol{r}_i$ and $\boldsymbol{r}_{i+1}$ is $\frac{\Delta s}{2}$. Next, the normalized field $\tilde{\boldsymbol{u}}$ is calculated on positions $\boldsymbol{r}_0$, $\boldsymbol{r}_2$,...$\boldsymbol{r}_i$, $\boldsymbol{r}_{i+2}$...by using RK which can be formulated as

$$\begin{cases} \boldsymbol{k}_1 &= -\{\tilde{\boldsymbol{u}}(\boldsymbol{r}_i)\cdot\frac{\boldsymbol{D}(\boldsymbol{r}_i)}{\tilde{n}^2(\boldsymbol{r}_i)}\}\boldsymbol{T}(\boldsymbol{r}_i) \\ \boldsymbol{k}_2 &= -\{[\tilde{\boldsymbol{u}}(\boldsymbol{r}_i) + \frac{\Delta s}{2}\boldsymbol{k}_1]\cdot\frac{\boldsymbol{D}(\boldsymbol{r}_{i+1})}{\tilde{n}^2(\boldsymbol{r}_{i+1})}\}\boldsymbol{T}(\boldsymbol{r}_{i+1}) \\ \boldsymbol{k}_3 &= -\{[\tilde{\boldsymbol{u}}(\boldsymbol{r}_i) + \frac{\Delta s}{2}\boldsymbol{k}_2]\cdot\frac{\boldsymbol{D}(\boldsymbol{r}_{i+1})}{\tilde{n}^2(\boldsymbol{r}_{i+1})}\}\boldsymbol{T}(\boldsymbol{r}_{i+1}) \\ \boldsymbol{k}_4 &= -\{[\tilde{\boldsymbol{u}}(\boldsymbol{r}_i) + \Delta s\boldsymbol{k}_3]\cdot\frac{\boldsymbol{D}(\boldsymbol{r}_{i+2})}{n^2(\boldsymbol{r}_{i+2})}\}\boldsymbol{T}(\boldsymbol{r}_{i+2}) \\ \tilde{\boldsymbol{u}}(\boldsymbol{r}_{i+2}) &= \tilde{\boldsymbol{u}}(\boldsymbol{r}_i) + \frac{\Delta s}{6}(\boldsymbol{k}_1 + 2\boldsymbol{k}_2 + 2\boldsymbol{k}_3 + \boldsymbol{k}_4) \end{cases} \tag{4}$$

Based on the intensity law of geometric field tracing [5], the amplitude $||\tilde{\boldsymbol{E}}(\boldsymbol{r})||$ can be calculated from ini-

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tial condition ($||\tilde{\boldsymbol{E}}(\boldsymbol{r}_0)||$, $\hat{\boldsymbol{s}}(\boldsymbol{r}_0)$, $n(\boldsymbol{r}_0)$, $\sigma(\boldsymbol{r}_0)$) as

$$||\tilde{\boldsymbol{E}}(\boldsymbol{r})|| = ||\tilde{\boldsymbol{E}}(\boldsymbol{r}_0)|| \cdot \sqrt{\frac{\hat{s}_z(\boldsymbol{r}_0)\cdot n(\boldsymbol{r}_0)\cdot\sigma(\boldsymbol{r}_0)}{\hat{s}_z(\boldsymbol{r})\cdot n(\boldsymbol{r})\cdot\sigma(\boldsymbol{r})}} \quad (5)$$

where $\hat{\boldsymbol{s}}(\boldsymbol{r})$ is the ray direction at position $\boldsymbol{r}$, which is defined as $\hat{\boldsymbol{s}}(\boldsymbol{r}) = \frac{\boldsymbol{T}(\boldsymbol{r})}{n(\boldsymbol{r})}$ and $\sigma(\boldsymbol{r})$ denotes the cross section of the ray tube.

Finally, electric field value $\tilde{\boldsymbol{E}}(\boldsymbol{r})$ along the ray path is calculated as follows:

$$\tilde{\boldsymbol{E}}(\boldsymbol{r}) = ||\tilde{\boldsymbol{E}}(\boldsymbol{r})||\tilde{\boldsymbol{u}}(\boldsymbol{r})\exp(ik_0 OPL(\boldsymbol{r})) \quad (6)$$

After ray path and field along ray are calculated, spot diagram and field value on spots are obtained in detector plane. However, in the concept of unified field tracing [5], fields on spot diagram do not match the requirement. Here we use the mesh concept in unified field tracing to achieve a continuous field in detector plane.

3 Examples

Here we model an optical system, where a VCSEL source is coupled into a multimode optical fiber, whose core has a 50 μm diameter, as shown in Fig. 1

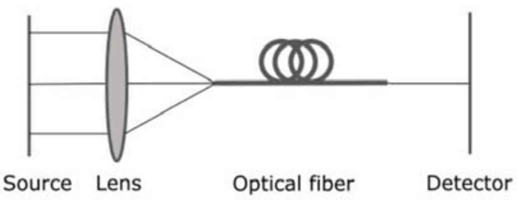

Fig. 1 *Sketch of the optical system with an optical fiber.*

The fiber is modeled by split-step method (SSM) and geometric field tracing (GFT), respectively. Fields in xz plane and in detector plane (xy) plane are shown in Fig. 2.

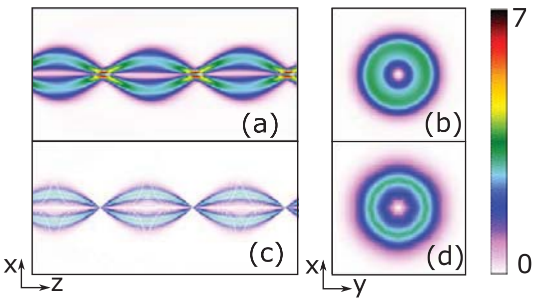

Fig. 2 *Amplitude of E_x component of the electromagnetic field in fiber:(a) field in xz plane, which is calculated by split-step method; (b) field in detector plane calculated by split-step method; (c) field in $x-o-z$ plane, which is calculated by geometric field tracing; (d) field in detector plane calculated by geometric field tracing*

Comparing the computing time of this example, SSM takes more than 1 hour but GFT just takes 5 minutes. GFT shows obvious advantage in computing time.

Comparing Fig. 2 (a) and (c), electromagnetic fields calculated by both algorithms are similar except in focus regions. This deviation comes from the approximation of geometrical optics. The coupling efficiency is calculated by dividing output power by input power, as shown in Tab. 1

	SSM	GFT
Input power (W)	1.986E-11	
Output power (W)	1.643E-11	1.736E-11
Coupling efficiency (%)	82.73	81.50

Tab. 1 *Coupling efficiency*

To compare the simulation results of polarization state by both approaches, we change the source into a linear polarization($E_x = 1, E_y = 0$) plane wave. After propagation in the same fiber, final field by GFT gives a small E_y, which is polarization crosstalk, while SSM cannot handle the crosstalk.

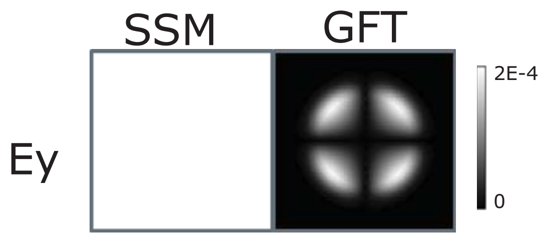

Fig. 3 *E_y calculated by SSM and GFT in detector plane.*

4 Conclusion

In this work, we propose a geometrical optics based approach to model graded-index media, which works efficiently and takes care of the field properties. Compared to spit-step method, it gives more accurate results of polarization but neglects diffraction because of the approximation of geometrical optics.

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