Theoretical description of a method for simultaneous measurement of phase and arbitrary polarization

C. Kellermann, S. Rothau, N. Lindlein

Universität Erlangen-Nürnberg, Institut für Optik, Information und Photonik, Erlangen
christine.kellermann@physik.uni-erlangen.de

MOTIVATION

- New interferometric measurement for simultaneous determination of arbitrary phase and polarization distributions of an object wave
- Extension of phase shifting interferometry (PSI) to polarization and phase shifting interferometry (P&PSI) with given equidistant polarization and phase steps
- Different polarization steps allow calculation of object polarization

Intensity pattern (analog to Michelson interferometric formula):

\[ J(x, y) = \frac{E_{1}(x, y) E_{2}(x, y)}{E_{10} E_{20}} e^{i \phi(x, y)} \]

with \( \phi' = \phi - \delta_x, \delta = \delta_y - \delta_x \), and \( \alpha = \arctan(\frac{\delta_y}{\delta_x}) \)

Object beam: phase \( \Phi \) and polarization states described by \( \alpha \) and \( \delta \):

\[ J_{O}(x, y) = E_{O}(x, y) e^{i \Phi(x, y)} \]

Reference beam: In the setup the different polarization states are set by a combination of a half-wave plate (HWP) and a quarter-wave plate (QWP).

\[ J_{R}(x, y) = E_{R}(x, y) e^{i \Phi(x, y)} \]

It’s appropriate to characterize the reference Jones vector by the orientation \( \Psi \) and the ellipticity \( \chi \) of the polarization ellipse:

\[ J_{R} = E_{R}(x, y) e^{i \Phi(x,y)} \]

Intensity pattern (analog to Michelson interferometric formula):

\[ I = |J_{O} + J_{R}|^2 = I_{0} \left[ 1 - \frac{1}{2} \left( \cos(\alpha) \left[ \cos(\Phi - \varphi) \cos(\Psi + 2\chi) - \cos(\Psi) \sin(\Phi - \varphi) \right] + \sin(\alpha) \left[ \sin(\Psi + 2\chi) \cos(\Phi - \varphi) + \sin(\Psi) \sin(\Phi - \varphi + \delta) \right] \right) \right] \]

MEASUREMENT METHOD

- The intensity \( I_{j} \) is measured for each polarization and phase step (\( \Psi, \chi, \varphi \)).
- Rewrite the intensity into a vector product

\[ I_{j} = \bar{Y}^{T} \bar{X}_{j} \]

- Thereby \( \bar{Y} \) consists of all unknown parameters \( (I_{0}, \Phi, \alpha, \delta) \) whereas \( \bar{X}_{j} \) includes the adjustable reference values \( (\Psi, \chi, \varphi) \)

- With the matrix \( \bar{A} \) the equation can be solved for \( \bar{Y} \):

\[ \bar{Y} = \bar{A}^{-1} \bar{X}_{j} \]

Only for specified values of \( \psi_{j} \in [0, \pi], \chi_{j} \in [-\pi/4, \pi/4], \varphi_{j} \in [0, 2\pi] \) \( \bar{A} \) is invertible.

- Different evaluation algorithms are possible. For each the polarization and phase steps are equidistant and there must be at least \( N \geq 3 \) images.

  - “Non averaging” algorithm: phase steps: \( \delta_{x} = \frac{\pi}{2N_{x}}, \) possible for \( N_{x} \geq 2 \) and \( N_{y} \geq 3 \)
  - “Averaging” algorithm: phase steps: \( \phi_{x} = \frac{\pi}{N_{x}}, \) possible for \( N_{x} \geq 2, N_{y} \geq 3 \) and \( N_{y} \geq 4 \)

MEASUREMENT SETUP

- Folded Mach-Zehnder interferometer allows almost normal (5°) incidence on the mirrors, so influences on the polarization are suppressed.
- With a Wollaston prism as beamsplitter (BS) the intensity ratio between object and reference arm is adjustable and therefore the visibility of the interferogram can be improved.
- Polarization shifting is enabled by rotating the HWP and QWP in the reference arm. For phase shifting mirror M1 in the object arm is moveable.
- To conserve the incident polarization distributions a special phase grating is used as beam combiner (BC).