Influence of Selection of Discrete Set of Points on Calculation of Correlation Coefficient in Processing of Image Data

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This work describes an analysis of the calculation of correlation coefficient in processing of image data. The correlation analysis is then used for a quantitative evaluation of interferograms in optical testing.

1 Introduction

A correlation coefficient is a reliable parameter for a comparison of digital images and correlation techniques find applications in various parts of science and engineering.

We perform an analysis of the calculation of a correlation coefficient between two functions (images) which are defined in a specified area on a discrete set of points. The influence of the number of points and their distribution in the specified area on the calculation of the correlation coefficient is investigated. The results of this analysis were used in a method for evaluation of interferograms, which can be in principle used for measurements of the shape of optical surfaces, where the correlation coefficient is used as a measure of the similarity of interference patterns.

2 Correlation coefficient

Suppose that we have two sets of data (digital images), e.g. two interferograms I_1 and I_2. The correlation coefficient R_{12} between two data sets is defined as

$$R_{12} = \frac{\operatorname{cov}(I_1, I_2)}{\sqrt{\operatorname{var}(I_1)\operatorname{var}(I_2)}}, \quad R_{12} = \langle -1, 1 \rangle . \quad (1)$$

where var(I_1) and var(I_2) denote the variances of the functions I_1 and I_2, and $\operatorname{cov}(I_1, I_2)$ is the covariance.

The correlation coefficient on a discrete set of 2D image data (between image I_1 and I_2) can be calculated using the following formula

$$R_{12} = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} \Delta I_1(x_i, y_j)\,\Delta I_2(x_i, y_j)}{\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N}\left[\Delta I_1(x_i, y_j)\right]^2}\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N}\left[\Delta I_2(x_i, y_j)\right]^2}}, \quad (2)$$

where indices (*i,j*) define individual pixels of images with coordinates (x_i, y_j), *M* and *N* define the size of the image, values $\overline{I_1}$ and $\overline{I_2}$ denote mean values and

$$\Delta I_1(x_i, y_j) = I_1(x_i, y_j) - \overline{I}_1,$$
$$\Delta I_2(x_i, y_j) = I_2(x_i, y_j) - \overline{I}_2. \quad (3)$$

The discretization of 2D area into a set of points, from which the correlation coefficient is calculated, affects the value of the correlation coefficient. We analyzed different types of the selection of points in 2D area of a circle (rectangular grid, radially distributed points, randomly distributed points) - Fig.1. We performed an analysis on correlation of special 2D data - interferograms (Fig.2).

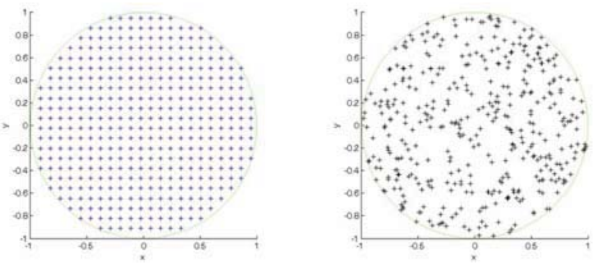

Fig. 1 *Rectangular uniform grid and randomly distributed grid.*

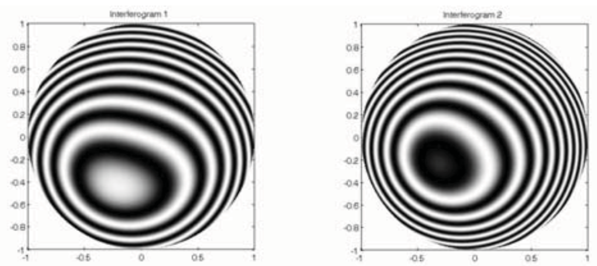

Fig. 2 *Interferograms for analysis.*

The results of the analysis for different types of discretization grid are given in Table 1.

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grid of n^2 points	rectangular grid	radial grid	random grid
n = 20	0.2951	0.2865	0.3288
n = 50	0.2858	0.2859	0.2743
n = 100	0.2838	0.2858	0.2736
n = 200	0.2855	0.2858	0.2818
n = 1000	0.2858	0.2858	0.2865

Tab. 1 *Correlation coefficient for different grids and number of grid points*

3 Application to interferogram testing

The correlation coefficient between two interference patterns may serve as a very robust parameter for the determination of the similarity of interferograms, which is very sensitive even to small deviations between shapes of interference fringes. One can use it for the quantitative comparison of interferograms and evaluation of shape deviations of optical surfaces in interferometric testing.

We used correlation for the quantitative evaluation of the fringe pattern during the interferometric testing of the shape of optical surfaces. The method uses the correlation coefficient as the measure of the similarity between the virtual (nominal) interferogram, which corresponds to the nominal shape of the tested surface, and the measured interferogram, which corresponds to the fabricated optical surface that is interferometrically tested.

The principle of the evaluation method is schematically shown in Fig. 3.

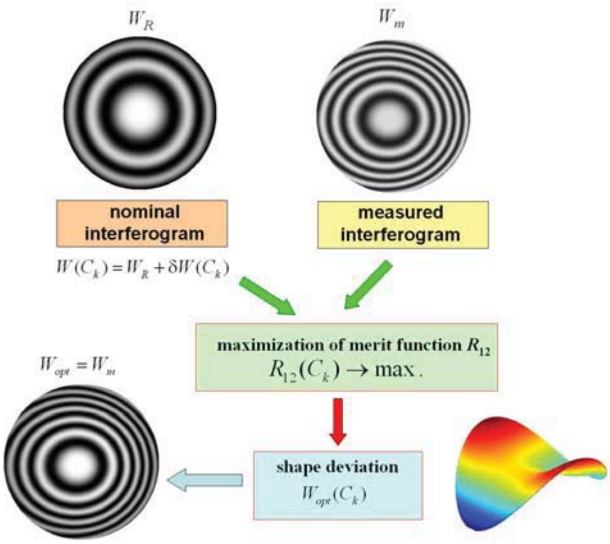

Fig. 3 *Principle of correlation analysis for quantitative interferogram evaluation.*

The evaluation method compares the shape of measured interferogram with the nominal shape by means of the optimization of the correlation coefficient value. That means to find such function $\delta W(x,y)$, when both interferograms will be the most similar, i.e. when the fringe shape is identical in both interferograms. This case corresponds to the maximum value of the correlation coefficient between interferograms. Then, it holds that the optical path difference in both interferograms is the same, i.e. $W_m = W_R + \delta W$. The function δW can be, for example, described by Zernike polynomial coefficients C_k. One can calculate very precisely deviations of the tested surface shape from the nominal shape by the mentioned principle.

4 Conclusions

We analyzed the functionality of the described evaluation method on various types of real interferometric measurements and compared the obtained results to measurements with the Zygo interferometer GPI/XP. Figure 4 presents the interferogram of the tested flat surface (Fig. 4 left), nominal interferogram (Fig. 4 middle) and the corresponding interferogram (Fig. 4 right) after the optimization process.

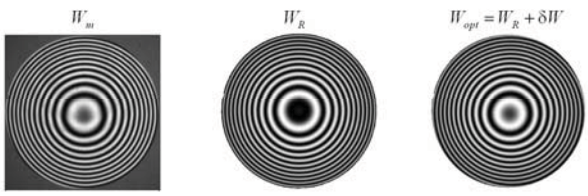

Fig. 4 *Interferograms of tested surface.*

The corresponding calculated optical path difference is shown in Fig. 5. The results were compared to the measurement using the Zygo interferometer (Fig. 5 right).

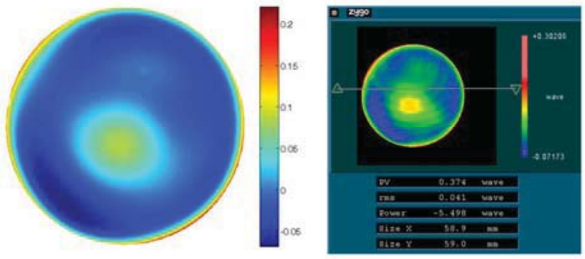

Fig. 5 *Calculated optical path difference.*

One can see that measurement results are very similar (correlation method: rms 0.041λ, Zygo MetroPro: rms 0.041λ, piston, tilt and defocus removed).

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