Light Propagation through Uniaxial and Biaxial Crystals: A Fully Vectorial Simulation Technique

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We present fully vectorial simulation approaches based on field tracing concept for light propagation through optically anisotropic media: the diffractive field tracing based on spectrum-of-plane-wave decomposition, and the geometric field tracing based on local-plane-wave approximation.

1 Introduction

Optical components made out of anisotropic crystals are widely used in modern optical systems, as in Fig. 1, and they play important roles in polarization control/manipulation. To enable simulations of such complicated systems, we need to investigate the propagation of light through anisotropic media with the polarization effects taken into consideration. Based on the field tracing concept [1], we propose two approaches to propagate electromagnetic fields through anisotropic media.

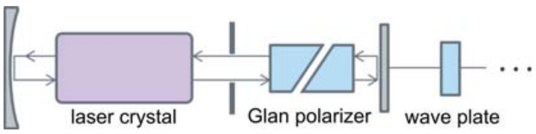

Fig. 1 *An optical system containing multiple components that are made out of anisotropic media.*

In field tracing, a complicated system is divided into subdomains and appropriate simulation techniques are used in different subdomains. We define a subdomain that contains the anisotropic medium (referred to as crystal in the following) which is optically specified by a 3×3 dielectric tensor $\bar{\epsilon}$, as shown in Fig. 2.

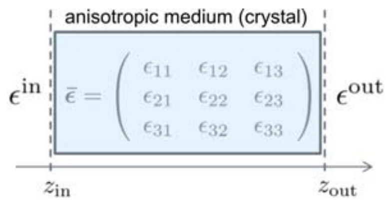

Fig. 2 *A subdomain containing an anisotropic medium.*

It is assumed that the light enters the crystal from an isotropic medium with dielectric constant ϵ^{in} and exits the crystal into an isotropic medium with ϵ^{out}. Then the simulation task can be formulated as

$$\begin{pmatrix} V_1^{\text{out}}(x, y, z_{\text{out}}) \\ V_2^{\text{out}}(x, y, z_{\text{out}}) \end{pmatrix} = \boldsymbol{\mathcal{C}}_{\text{aniso}} \begin{pmatrix} V_1^{\text{in}}(x, y, z_{\text{in}}) \\ V_2^{\text{in}}(x, y, z_{\text{in}}) \end{pmatrix}, \tag{1}$$

where we use $\boldsymbol{V} = (V_1, V_2, V_3, V_4, V_5, V_6)^{\text{T}} = (E_x, E_y, E_z, H_x, H_y, H_z)^{\text{T}}$ to represent the electromagnetic field, and it has been shown in [1] that in homogeneous isotropic media it is sufficient to use only two field components to characterized the fully vectorial electromagnetic field. To find the operator $\boldsymbol{\mathcal{C}}_{\text{aniso}}$ in Eq. (1) is our task.

2 Diffractive field tracing

By using Fourier transform the input field $V_\ell(x, y, z_{\text{in}})$, with $\ell = 1 \text{ or } 2$, can be decomposed into plane waves. Then the remaining task it to propagation each incident plane wave through the crystal and that includes refraction of plane wave at crystal surface(s) and propagation inside it. To solve these questions, the fundamental modes of electromagnetic fields in the crystal have to be known. It is shown in [2] that plane wave solutions exist in crystals. Following [2] we formulate Maxwell's equations in a 4×4 matrix form. By using the following plane wave ansatz

$$\boldsymbol{V}(\boldsymbol{r}) = \breve{\boldsymbol{V}} \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{r}), \tag{2}$$

an eigenvalue problem can be obtained

$$\frac{k_z}{\omega} \begin{pmatrix} \breve{V}_1 \\ \breve{V}_2 \\ \breve{V}_4 \\ \breve{V}_5 \end{pmatrix} = \bar{\Omega} \begin{pmatrix} \breve{V}_1 \\ \breve{V}_2 \\ \breve{V}_4 \\ \breve{V}_5 \end{pmatrix}, \tag{3}$$

where $\bar{\Omega}$ is a 4×4 matrix. For a given spatial frequency pair (k_x, k_y), all the elements of $\bar{\Omega}$ is known and solving Eq. (3) yields four solutions.

Based on the knowledge of the plane wave solutions inside the crystal, the rule of refraction rules at plane interface between the isotropic medium and the crystal can be derived. For each incident plane wave, the given spatial frequency (k_x, k_y) must by conserved at the interface. Using this (k_x, k_y) in Eq. (3) four plane wave solutions can be obtained and by examining the Poynting vectors we find that only two of them propagate in the $+z$-direction. By matching the field values at the boundary, a 4×4 coefficients matrix can be found to calculate the amplitudes of the transmitted and reflected plane waves [3].

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Applying the rule of refraction on the front surface, we obtain the transmitted plane waves inside the plane waves inside the crystal and the propagation of the plane waves are governed by Eq. (2). On the rear surface the refraction rule is used again to calculate the output field.

3 Geometric field tracing

The SPW approach handles diffraction effects rigorously by taking all the spatial frequencies into account but on the other side it requires a full sampling of the complex-valued field on certain planes. Sometimes a huge amount of sampling efforts are needed, while the diffraction effects are not of much interest. As a matter of fact, the behavior of light is dominated by geometrical optics and in such cases we introduce the following ansatz

$$\boldsymbol{V}(\boldsymbol{r}) = \tilde{\boldsymbol{V}}(\boldsymbol{r}) \exp[\mathrm{i}\phi(\boldsymbol{r})] \tag{4}$$

The geometrical optics approximation can be understood as that the variation of $\tilde{\boldsymbol{V}}(\boldsymbol{r})$ is so small in comparison to that of $\phi(\boldsymbol{r})$ and can be neglected. Substutite the ansatz in Eq. (4) into Maxwell's equations and apply the approximation, we can conclude that locally the field behaves in the same way as a plane wave. Based on this conclusion, all the discussions in the previous section about plane waves can be directly applied in geometric field tracing as well. As is written in Eq. (4), the field can be sampled as two parts in geometric field tracing and that brings numerical advantages without losing information on, for example, refraction and polarization.

4 Example

Using the optical design software VirtualLab Fusion [4], we perform simulation of conical refraction in a biaxial naphthalene crystal, with $n_1 = 1.525$, $n_2 = 1.722$ and $n_3 = 1.945$ @ 633 nm [5]. A slightly focused, circularly polarized Gaussian field with 100 um waist radius is used as the input and it propagates along the optical axis of the biaxial crystal. The output field at the rear surface of the crystal is calculated by using both the diffractive field tracing approach and the geometric field tracing approach.

As is known from the experiments in [5] and seen from Fig. 3, after passing through the biaxial crystal along its optial axis the field evolves into two bright rings with a dark region in between. Sharp edges can seen between two rings in Fig. 3(b), because diffraction is not included in geometric field tracing. From another aspect, about 75,000 sampling points in two dimensions are used in the diffractive field tracing approach in Fig. 3(a), while only around 2500 rays are used to produce the result in Fig. 3(b) using the geometric field tracing approach.

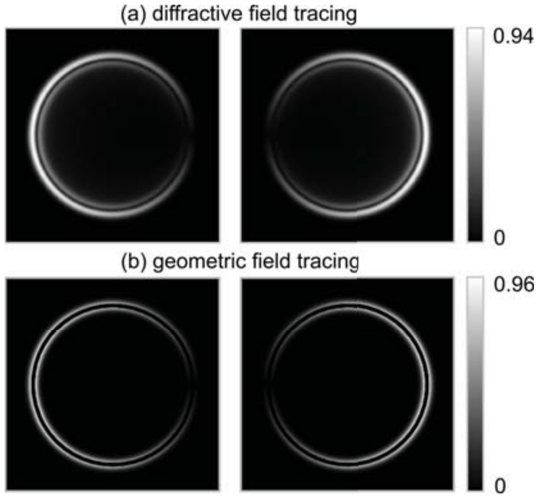

Fig. 3 *Amplitudes of output field:* $|V_1^{\mathrm{out}}(x, y, z_{\mathrm{out}})|$ *(left) and* $|V_2^{\mathrm{out}}(x, y, z_{\mathrm{out}})|$ *(right) obtained by using (a) diffractive field tracing and (b) geometric field tracing.*

5 Summary

Two vectorial approaches for electromagnetic field propagation through anisotropic media are developed based on the concept of field tracing. Together with existing field tracing techniques [1, 6], it enables the simulation of complex system containing elements that are made of uni-/biaxial crystals. An example of application can be found in [7].

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