Modified spectrum of plane wave method by adding perfectly matched layers to eliminate aliasing

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We propose a method to add Perfectly Matched Layers(PMLs) at the lateral sides of the computation window by complex coordinate transformation. It modifies the Helmholtz equation from which the rigorous numerical solution for Spectrum Plane Wave(SPW) method is derived. The numerical experiment results are compared with the traditional SPW, which achieve nice agreement. It shows that the SPW method with PMLs(SPW_PMLs) performs well when simulating the highly divergent beams.

1 Introduction

In optical modelling, the simulation of divergent beam propagation in free space with Spectrum of Plane Wave(SPW) is often required in the field of digital holography[1][2] and field tracing[3][4].However, when the size of the computation window is fixed, aliasing often occurs. How to avoid the aliasing with the fixed window size is the issue.Band-limited SPW was proposed to avoid aliasing[5][6], high frequency filter is implemented to limit bandwidth of the transfer function. However, when the cutting frequency is less than the maximum frequency of the field. The calculation is not accurate because the high frequencies are lost.
We propose a method adding Perfectly Matched Layers(PMLs)[7] to SPW at the lateral sides of the computation window as shown in Fig. 1. Fields of all frequencies even the evanescent ones could be absorbed in the PMLs region, therefore the results are accurate.

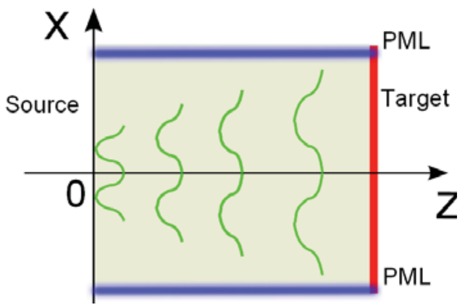

***Fig. 1** Schematic of the wave propagation in free space*

2 Methods

2.1 Basic Formulation

We take a TE polarization(electric field is y-direction which is perpendicular to the paper) field with y-invariant for example. It is expanded into Fourier Series.

$$E(x,z) = \sum_{m=-\infty}^{\infty} s_m(z)\exp(-ik_{xm}x), \quad (1)$$

with $k_{xm} = k_{x0} + m\frac{2\pi}{\Lambda}, m \in Z$. But when PMLs are implemented, it satisfies the modified Helmholtz equation:

$$\frac{1}{f'(x)}\frac{\partial}{\partial x}\left(\frac{1}{f'(x)}\frac{\partial}{\partial x}\tilde{E}_y(x,z)\right) + \frac{\partial^2}{\partial z^2}\tilde{E}_y(x,z) + k_0^2\epsilon\tilde{E}_y(x,z) = 0, \quad (2)$$

we expanded $1/f'(x)$ into Fourier series as well:

$$\frac{1}{f'(x)} = \sum_{m==-\infty}^{\infty}\hat{f_m}\exp\left(i\frac{2\pi m}{\Lambda}x\right). \quad (3)$$

Inserting Eq. 1 and Eq. 3 into Eq. 2, and retaining 2M+1 truncation orders. we could get:

$$\frac{d^2}{dz^2}\hat{\mathbf{E}}(z) + \mathbf{A}\hat{\mathbf{E}}(z) = 0, \text{with } \mathbf{A} = \left(k_0^2\epsilon\mathbf{I} - (\mathbf{F}\mathbf{K}_x)^2\right), \quad (4)$$

where $\mathbf{I}$ is the identity matrix. $\mathbf{K}_x$ is a diagonal matrix with k_{xm} on its diagonal. $\mathbf{F}$ is the Toeplitz matrix of the Fourier coefficient of $1/f'(x)$. It has the solution

$$\hat{\mathbf{E}}(z) = \mathbf{W}\exp(-i\mathbf{Q}z)\mathbf{c}^+, \quad (5)$$

where $\mathbf{Q}$ is the square root of the eigenvalue of the matrix $\mathbf{A}$. $\mathbf{W}$ is the corresponding eigenvector.

2.2 Approximation

Since PMLs are only at both lateral sides of the computational window, the matrix $\mathbf{A}$ has most of the valued entries near the diagonal. Therefore, we could approximate the matrix $\mathbf{A}$ by the block diagonal matrix $\mathbf{B}$. The eigenvalue and eigenvector of $\mathbf{B}$ is the combination of these of $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$,.... . With the approximation, the complexity of the eigensolver process could be reduced.

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2.3 Criteria to perform the SPW_PMLs

In order to save numerical effort as well as obtained the field in the desired region, the criteria is shown in this section. By the fast Fourier Transformation, the time complexity and space complexity of SPW are:$O((2N+1)\log(2N+1))$ and $O((2N+1))$ respectively. where $2N+1$ is the sampling points of SPW. In the meanwhile, the time complexity and space complexity of SPW_PMLs are $O((2N+1)^3)$ and $O((2M+1)^2)$ respectively. Then we could obtain the criteria from the time complexity and space complexity as:

$$\frac{\Lambda_{\text{SPW}}}{\Lambda_{\text{PMLs}}} > \sqrt{\frac{(2N+1)^2}{\log(2N+1)}}^{1/3} \tag{6}$$

$$\frac{\Lambda_{\text{SPW}}}{\Lambda_{\text{PMLs}}} > \sqrt{\frac{(2N+1)}{\log(2N+1)}}^{1/2} \tag{7}$$

3 Numerical experiment results

Highly divergent Gaussian wave with the half divergent angle $\theta = 60°$ and wavelength of $\lambda = 532$ nm is normally incident to the free space surrounded by air with the refractive index $n = 1.0003$. Note that in this case, evanescent wave is included. The incident wave is TE polarized. The propagation distance is $d = 15$ µm. For SPW simulation, the window size is 10mm, sampling points are 800001 and 200 in x and z direction respectively. For SPW_PMLs simulation, the window size is 12 µm, sampling points are 405 and 200 in x and z direction respectively. The width of PMLs is 1.2 µm. For approximated SPW_PMLs simulation, the window size is 12 µm, sampling points are 405 and 200 in x and z direction respectively. The width of PMLs is also 1.2 µm. The number of blocks is 5; The results are show in figure. 2, 3, 4,

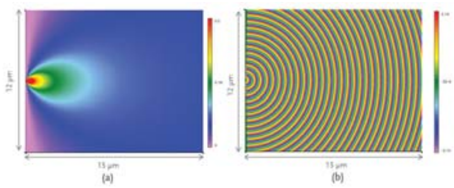

Fig. 2 *Results from SPW (reference field) in x-z plane. Computational time is 18.6 s. (a)Amplitude of the field.(b)Phase of the field.*

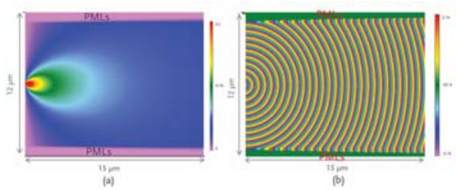

Fig. 3 *Results from SPW_PMLs in x-z plane. Computational time is 10.13 s. (a)Amplitude of the field.(b)Phase of the field.*

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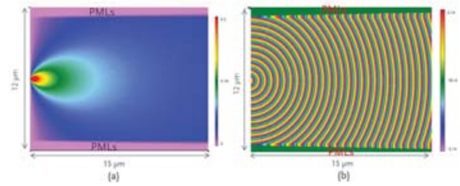

Fig. 4 *Results from approximated SPW_PMLs in x-z plane.Computational time is 0.97 s. (a)Amplitude of the field.(b)Phase of the field.*

Convergent investigation is also performed for the SPW_PMLs, we first define the deviation as:

$$D_{\text{dev}} := \frac{\sum_{x,y} |V_{\ell,\text{Analysis}}(x,y,z) - V_{\ell,\text{Reference}}(x,y,z)|^2}{\sum_{x,y} |V_{\ell,\text{Reference}}(x,y,z)|^2} \tag{8}$$

The reference field is the results from SPW. The convergence test result shows that when the sampling points increase to $2M+1 = 225$, the deviation drops below 10^{-5}. When the sampling points increase to $2M+1 = 445$, the deviation drops below 10^{-6}.

4 Conclusion

The SPW_PMLs method is proposed to eliminate aliasing by adding PMLs through complex coordinate transformation. The approximated SPW_PMLs is also proposed to save numerical effort approximating the eigenvalue matrix by block diagonal matrix. The proposed method is validated by the numerical experiment. It could be used in the near field propagation to evaluate the evanescent wave behaviour in the near field.

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