Aliasing-free speckle propagation: Critical sampling rules

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In this contribution we clarify the topic of critical sampling for the numerical propagation of an optical wavefield. With this numerical framework we examine the problem of the iterative phase retrieval algorithm.

1 Introduction

One big challenge in wavefront sensing is to reveal the curvature of a wavefront from measured intensities. Besides interferometric methods, structured illumination or the illumination of modulating gratings we apply in this contribution the method of phase retrieval, i.e. the capturing of intensity images serving as a data base for an iterative wavefront reconstruction algorithm. Even though there are many contributions presenting successful iterative phase retrieval algorithms the numerical conditions, especially the representation of the field distributions and its propagation is still an open issue.[1]

2 Phase Retrieval Setup

Fig. 1 shows a typical setup for iterative phase retrieval.

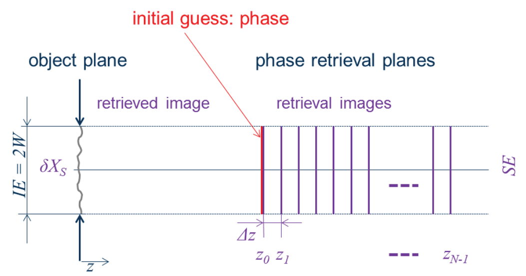

***Fig. 1** Typical setup for phase retrieval.*

A transparent object is illuminated by a plane wavefront of coherent light. The resulting wavefield propagates along z direction and is captured by a camera sensor as intensity images at different distances. These recorded images serve as a data set for the following phase retrieval algorithm. We start with an initial guess of a phase distribution, combine it with the amplitude of the first image, propagate numerically to the next retrieval plane, replace its amplitude by the measured one and propagate forward plane by plane until the most distant plane and then backwards to the first retrieval plane. We repeat this iteration cycle until we meet an abortion criterion, such as number of iterations, convergence, etc. Finally we propagate back to the object plane and get the complex distribution of the revealed optical wavefield.

3 Numerical Propagation and Test

The numerical propagation is the key operation of the phase retrieval algorithm. Based on the Kirchhoff diffraction integral, the output distribution in a diffraction plane at a propagation distance of z is determined by the convolution of the complex distribution of the object and the impulse response of free space. Here we assume Fresnel approximation. This diffraction integral can be processed either in spatial domain, called direct method (DM) or in spectral domain, called spectral method (SM). Even though both methods implement the same physical phenomenon, their results differ in general. For DM the spatial output extent SE along a lateral coordinate x equals to $SE = \lambda z/\delta X$, with the wavelength λ, the propagation distance z and the sampling rate of δX. For the SM the spatial extent SE remains the same as the input extent IE in object plane following $SE = IE = N\cdot\delta X$, with N as the number of sampling points. The critical sampling condition we get when we unify both equations of the SE's:

$$SE = IE = \frac{\lambda z}{\delta X} = N \cdot \delta X' \qquad (1)$$

Then, both methods, DM and SM, result in identical complex distributions and in identical sampling, i.e., ideal sampling [2]. For a given distance z the ideal sampling is adopted by either

- upsampling with a reduced sampling rate $\delta X'$ and unmodified lateral SE,IE or
- upscaling with an unchanged sampling rate δX and an expanded SE,IE by zero padding.

Now we are in a position to perform a numerical test for a forward propagation from the object to the farthest retrieval plane. First we have to expand the output extent to regard the diffraction of our object field.

It turns out that we have to ensure the ideal sampling condition first for the propagation from the object to the farthest retrieval plane (dashed violet

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line, s. Fig. 2) and afterwards for the individual propagation steps object – first retrieval plane as well as between the retrieval planes (dashed green lines). This prevents the lateral walking off of the wavefield and related aliasing artefacts. Even speckle fields don't walk off anymore under this regime. A correctly working numerical propagation reflects the reality and is a prerequisite for a reliably running iteration algorithm as well as its convergence [2].

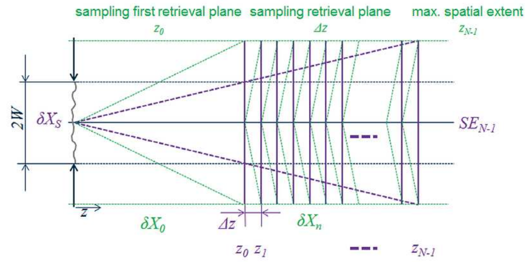

Fig. 2 *Scheme of the numerical propagation regarding ideal sampling and preventing the lateral walk off of the propagating optical wavefield.*

4 Experiments

After clarifying the numerical conditions we present experiments of phase retrieval. First we evaluate the influence of the initial guess of phase: random phase 1, random phase 2 and homogeneous (flat) phase distribution on an iris diaphragm as test sample.

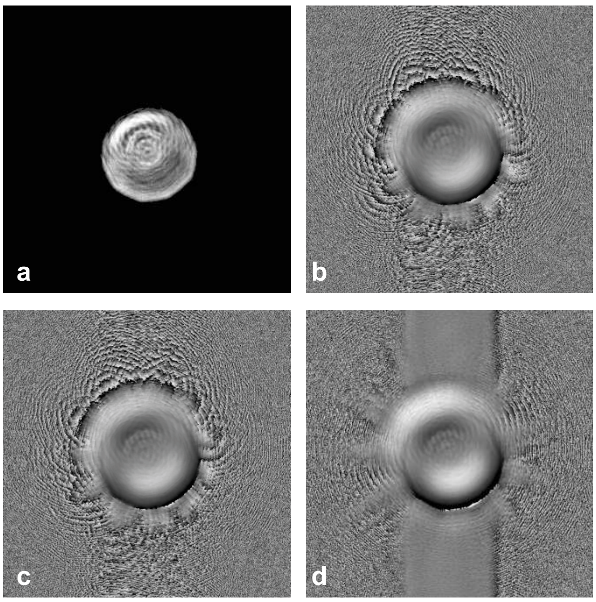

Fig. 3 *Influence of the initial guess of phase. a: retrieved intensity, b-d: retrieved phase by initial random phase 1 (b), 2 (c) and flat phase(d).*

It turns out that speckle distributions are not suited for observation because of their very quickly changing phase and intensity. The resulting distributions hardly show similarities. However for smooth wavefronts the revealed phase distributions are well approximated especially within the aperture area where the intensity is not zero. The results are shown in Fig. 3.

In a second experiment with a diffractive lens as test sample we observe the influence of the maximum lateral wavefield extent related to the size of most distant retrieval plane. There we've found out that the retrieval algorithm results in reasonable phase distributions when the wavefield is truncated, especially at the farthest retrieval plane. This is realized by adjusting an aperture stop of $2W$ in the object plane. Then the whole information is included within a cone of an axially propagated wavefield and external information does not corrupt the algorithm.

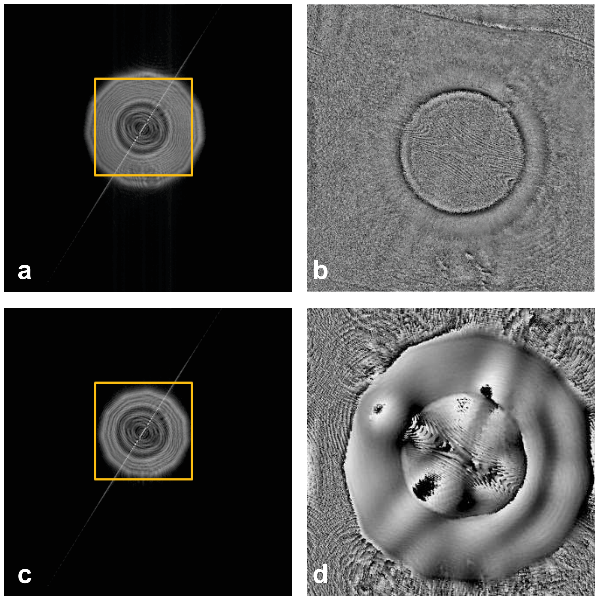

Fig. 4 *Truncation of the output wavefield at farthest plane related to the processed area (orange) by iterative phase retrieval. a: larger wavefield extent, c: smaller wavefield extent, b,d: retrieved phases.*

5 Conclusions

There are two necessary conditions for a successful phase retrieval algorithm: 1) obey the critical sampling condition for each propagation step and 2) consider the truncation of the maximal wavefield extent. Then, the volumetric intensity information of a propagated optical field can be used to reveal the complex wavefront of an illuminated object by an iterative algorithm.

References

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