Quantized tuneable Helix Phase Plates - Design and Experiment

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Light with a helical phase is used in multiple applications like optical tweezing [1] and resolution improvement in optical systems [2]. A tuneable helix phase plate based on Alvarez-Lohmann leses was earlier introduced by Bernet. A disadvantage of these elements is the generation of undesired phase sections. We demonstrate a quantization method that eliminates these effects.

1 Introduction

A helix phase plate transforms an incident plane wave into a spiral stair case wave front. In cylindrical coordinates (r, θ, z) the phase rises linearly in the angular coordinate θ while remaining constant along the radial coordinate r, generating a phase dislocation, also called vortex, at the center ($r = 0$). Light with helical phase is used in optical tweezers to rotate specimens due to the generated orbital angular momentum [1]. Light with helical phase will, when focussed with a lens, generate a ring shaped intensity distribution with zero intensity on axis. This is used e.g. in vortex coronagraphs to make dark objects in the vicinity of bright sources visible [2]. Changing the steepness of the helical phase, the so called topological charge l, allows one to tune the intensity ring diameter and the amount of angular momentum the light carries. Hence it it desirable to have a simple optical element enabling that tuning. For that purpose the Alvarez-Lohmann principle for generating tuneable optical elements was chosen.

2 Alvarez-Lohmann Principle

The addition of two phase functions differing only in sign results in no relative phase change since both functions cancel each other. When a lateral offset or shift between the initial phase functions is introduced, the new function resulting from the addition correlates with the first derivative of the initial functions with respect to the shift direction. Furthermore the resulting function is weighted by the amount of shift. This was used independently by Adolf Lohmann and Luis Alvarez to describe tuneable lenses [3, 4]. Harm and Bernet used this principle to generate diffractive tuneable helix phase plates [5, 6]. If the two diffractive elements are passed by light sequentially, they impose a helical phase onto it, which slope is proportional to the relative rotational shift between both elements. The initial functions of the elements are shown in eq.1.

$$\begin{aligned} \phi_1(r,\theta) &= l\theta^2 \\ \phi_2(r,\theta) &= -l\theta^2 \end{aligned} \tag{1}$$

$$\begin{aligned} &\phi_1(r,\theta+\Delta\theta)+\phi_2(r,\theta-\Delta\theta) \\ &= l((\theta+\Delta\theta)^2-(\theta-\Delta\theta)^2) \\ &= l\theta(4\Delta\theta) \end{aligned} \tag{2}$$

The resulting phase in eq.2 shows a linear dependency of θ, thus a helix phase plate, weighted by the constant l and twice the amount of relative shift $4\Delta\theta$.

3 Problem of inverse Phase Sections

A problem with Alvarez-Lohmann type tuneable optical elements tuned by rotation is the ambiguity of the rotational shift due to the 2π periodicity. As a result the phase function generated in an angular region the size of $2\Delta\theta$ is $\phi(r,\theta)_{res} = 4l\theta(\Delta\theta - 2\pi)$. This is depicted in principle in figure 1. As a result of these two inverse sections the intensity distribution in focus is an open ring, see figure 2 .

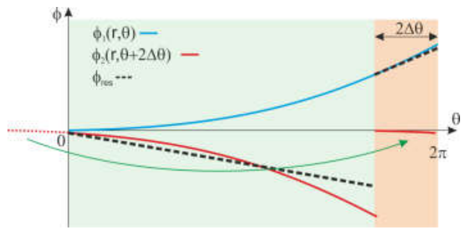

Fig. 1 *Generation of two inverse sections due to 2π ambiguity.*

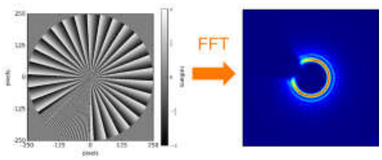

Fig. 2 *left: phase of two combines elements showing two sections of different slope; right: focussing the phase results in an open ring intensity distribution.*

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4 Quantization Method to eliminate inverse Sections

The effect of a second phase section can be eliminated for discrete tuning positions by quantizing the single Alvarez-Lohmann elements. A similar approach was proposed by Farn for lens arrays [7]. the condition for the quantization is, that if the resulting function is shifted by one period, here 2π, its phase yields an offset to the unshifted phase by an integer k multiple of one wavelength (2π), see eq.3.

$$\begin{aligned} \phi_1(r,\theta) &= \phi_1(r,\theta+2\pi) - 2k\pi \\ l\theta^2 &= l*(\theta^2+4\theta\pi+4\pi^2) - 2k\pi \\ k &= 2l(\theta+\pi) \\ 2l(-\pi) < k \le 2l(\pi) \; &for \; -2\pi < \theta \le 0 \end{aligned} \tag{3}$$

Now l can be chosen in these limits to realize equidistant quantization steps and to provide a feasible amount of tuning steps. For 200 tuning positions of the phase plate pair l is chosen as:

$$l = \frac{1}{2\theta_{inc}} \quad with\; \theta_{inc} = \frac{\pi}{100} \tag{4}$$

The diffractive elements are quantized according to eq. 5, where m is an integer number and $round$ represents a rounding operation to full integer values.

$$\begin{aligned} \phi_{1quant}(r,\theta) &= l\theta_{inc}^2(round(\frac{\theta+m\theta_{inc}}{\theta_{inc}})^2) \\ &= \frac{\theta_{inc}}{2}(round(\frac{\theta+m\theta_{inc}}{\theta_{inc}})^2) \end{aligned} \tag{5}$$

A detailed description of the quantization process can be found in [8].

5 Experimental Results

For feasible fabrication a second quantization step was applied to the elements to realize 4 level diffractive elements. The elements where illuminated with a plane wave of 500nm wavelength. After passing the elements, the light was focussed by a lens of 180mm focal length. The intensity distribution in focus was captured by a CCD for different tuning positions of the element pair. Figure 3 shows, that the simulated intensity is in good agreement with the measurement. The quantized elements realize a closed ring intensity distribution. The first major limitation of the method is the reduced efficiency of the elements quantized twice which is:

$$\eta = sinc^4(\frac{\pi}{level})sinc^2(\frac{m\theta_{inc}}{2}) \tag{6}$$

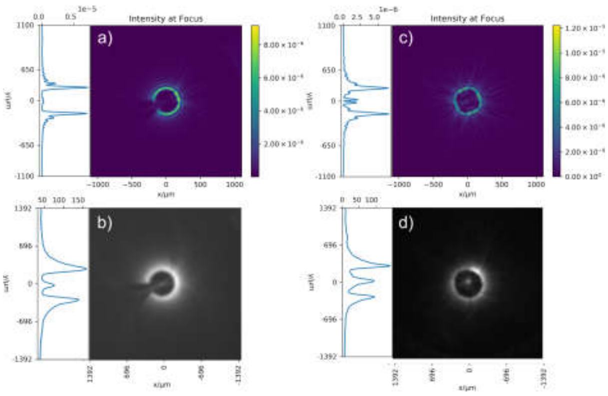

Fig. 3 *a: intensity simulation for non-quantized 4 level elements with 45° offset; b: measurement coresponding to a; c: intensity simulation for quantized 4 level elements with with 45° offset; d: measurement coresponding to c.*

A second limitation is the accuracy of the rotational tuning. Since the quantization holds only for integer tuning steps m of θ_{inc}, at rotation angles in between the inverse section is not eliminated. That is shown in figure 4 for 16 level elements.

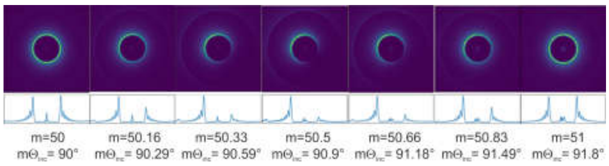

Fig. 4 *Intensity distribution for different tuning positions.*

6 Conclusion

Rotatory diffractive Alvarez Lohmann elements produce inverse sections upon tuning. These can be eliminated by quantizing the single phase plates radially [6] and/or angularly. The resulting elements are subject to the limitations discussed above and offer great potential for numerous applications.

References

[1] M. Padgett and L. Allen, "Light with a twist in its tail," Contemporary Physics **41**, 275–2858 (2000).

[2] G. Foo et al., "Optical vortex coronagraph," Optics Letters **30**, 3308–3310 (2005).

[3] A. Lohmann, British Patent 998,191 (May 29th 1964).

[4] L. W. Alvarez, US Patent 3,305,294 (Dec. 3rd 1964).

[5] W. Harm et al., "Adjustable diffractive spiral phase plates," Opt. Express **23**, 413–421 (2015).

[6] S. Bernet and M. Ritsch-Marte, "Adjustable refractive power from diffractive moiré elements," Applied Optics **47**, 3722–3730 (2008).

[7] M. W. Farn, "Agile beam steering using phased-arraylike binary optics," Appl. Optics **33**, 11070–11078 (1994).

[8] A. Grewe and S. Sinzinger, "Efficient quantization of tunable helix phase plates," Opt. Lett. **41**, 4755–4758 (2016).

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