

# Phase Retrieval Algorithms for Wavefront Analysis

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In this contribution we examine iterative methods for phase retrieval (PR) based on the conventional propagation algorithm on multiple planes (MPP) and the Simplified Extended Nijboer-Zernike Theory (SENZ). We investigate the stability and quality of these algorithms against tolerances in simulations and verify both methods on experimental data.

## 1 Introduction

The imaging quality of optical systems can be determined by the evaluation of wavefront aberrations within the exit pupil. Beside established methods for wavefront analysis we examine the iterative phase retrieval [1]. We illuminate the entrance pupil of an optical test system with a plane wave and capture a defocused image stack of intensities near the focal plane (see Fig. 1).

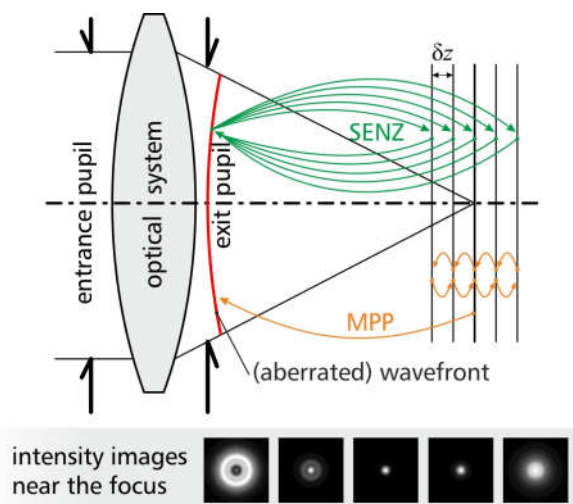


Fig. 1 Scheme of setup and phase retrieval algorithms.

## 2 Iterative phase retrieval algorithms

This image stack serves as a data set for the iterative PR algorithm. We first compose the complex wavefront by the measured amplitude and an initial guess of phase. Then this distribution is propagated to the next plane, the amplitude is replaced by the measured one, the phase is kept. After performing this procedure repeatedly from plane to plane through the image stack the achieved complex wavefield approaches to a convergent solution [1].

The MPP performs the numerical propagation based on the Kirchhoff diffraction integral (angular spectrum method). This method is affected by the restrictions of sampling. Following the concept of

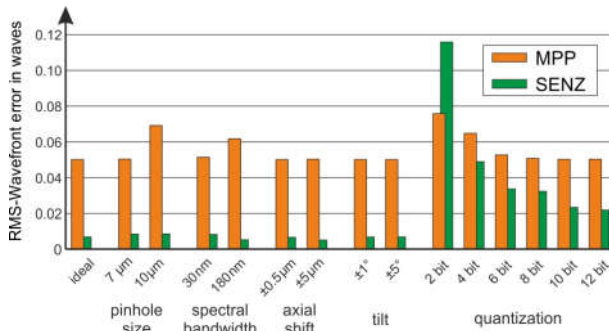
ideal sampling [4] there are only few constellations of parameters of pixel pitches, propagation distances and lateral extension of the retrieval planes possible which have to be defined before the measurement.

The SENZ algorithm is based on the decomposition of the complex wavefront into Zernike polynomials in the exit pupil. The distribution in the corresponding (image) focal plane is given by the Fourier transform. It is possible to introduce a small defocus to access the axial region around the focus [2],[3]. This enables an iterative phase retrieval algorithm on an image stack as mentioned above. The wave is propagated from one defocused plane to the next via the exit pupil (see Fig. 1). It has to take into account the trade-off between the numerical aperture and the defocus. A further challenge is the limitation of the number of Zernike coefficients for the wavefront description in order to keep the accuracy due to the discrete numerical representation within the PC. A proper choice depending on the aberrations to be expected is necessary in order to keep the algorithm convergent and stable.

The main challenge of measurement is the strong concentration of intensity in the focal plane. This has the consequence of a low lateral resolution due to finite pixel size and a reduced dynamic range of intensity caused by a low quantization of analog/digital converter (up to 12bits).

## 3 Simulations

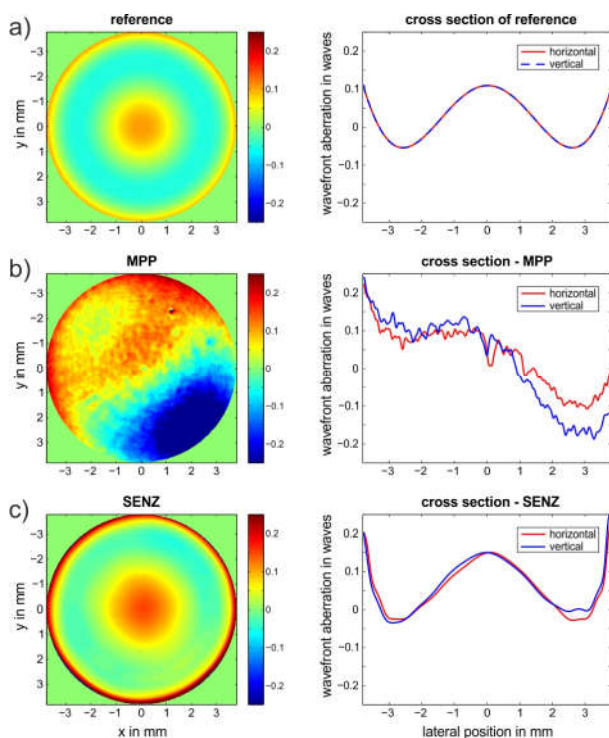
We examined the algorithms concerning the tolerances of illumination (pinhole diameter, spectral bandwidth), mechanical stages (axial and lateral lag, tilt) and the sensor (quantization) to the quality of the reconstructed wavefront in the exit pupil. The results are presented in Fig. 2. We see that (i) the MPP algorithm seems to be more sensitive to disturbances than the SENZ and (ii) the insufficient quantization of the analog-digital converter is the main source of errors in the the wavefront reconstruction.



**Fig. 2** Influence of tolerances on phase retrieval algorithms.

#### 4 Experimental results of an optical system

The MPP algorithm allows for a large defocus due to the sampling restrictions: here more than  $\pm 1000 \mu\text{m}$  relative to the focal plane for 7 axial positions. The illustrated example (standard deviation of 0.123 waves within 90% of exit pupil area) contains artifacts of noise and tilt (see Fig. 3b). The noise is caused by aliasing introduced by lateral nonlinearities of the spherical propagator kernel. The observed tilt can be considered as a mechanical misalignment.



**Fig. 3** Reconstructed wavefront of an optical system in exit pupil for a) reference without tolerance, b) MPP and c) SENZ.

For the SENZ algorithm the same optics is evaluated at a maximum defocus of  $\pm 100 \mu\text{m}$  for 6 measured planes. This method provides reliable results within the central region (standard deviation

of 0.1 waves within 90% of exit pupil area, see Fig. 3c). Near the pupil boundary the deviations increase strongly.

#### 5 Summary

The MPP algorithm and the SENZ method deliver reasonable reconstructed wavefronts based on measurements close to the focal region. Both methods require a high quantization of recorded intensities and a highly quasi single point illumination with a narrow spectral bandwidth for the measurement. The MPP algorithm is constrained by the ideal sampling criterion and therefore better suited for larger propagation distances (wider defocus range) between the image planes whereas the choice of an appropriate number of Zernike coefficients and a suitable defocus especially for high numerical apertures are critical for the SENZ method.

#### References

- [1] J. R. Fienup: „Phase retrieval algorithms a comparison“ in *Appl. Optics*, Vol. 21, No. 15, 1982, pp. 2758-2769
- [2] S. v. Haver, A. J. E. M. Janssen: „Advanced analytic treatment and efficient computation of the diffraction integrals in the extended Nijboer-Zernike theory“ in *Journal of the European Optical Society-Rapid publications* 8, 2013
- [3] Y. Wu, M. Hillenbrand, L. Zhao, S. Sinzinger, D. P. Kelly: „The Fresnel transform as a projection onto a Nijboer-Zernike basis set“ in *Optics Letters*, Vol. 40, 2015, pp. 3472-3475
- [4] D. G. Voelz, M. C. Roggemann: „Digital simulation of scalar optical diffraction revisiting chirp function sampling criteria and consequences“ in *Appl. Optics*, Vol. 48, No. 32, 2009, pp. 6132-6142