Improved reconstruction quality of holographic projections by controlling the individual pixel shape

Fabian Thiemicke*, Thorsten Klein*, Claas Falldorf*, Ralf B. Bergmann*,**

** BIAS-Bremer Institut für angewandte Strahltechnik GmbH, Klagenfurter Str. 2, 28359 Bremen, Germany*
*** University of Bremen, Faculty 01: Physics and Electrical Engineering and MAPEX, 28359 Bremen, Germany*

mailto:thiemicke@bias.de

We introduce a numerical approach to calculate computer generated holograms which use individually shaped pixels to improve the reconstruction quality compared to a hologram with a uniform pixel height.

1 Introduction

Holographic optical elements (HOE) are widely used for projection of desired intensity distributions, e.g. in security, lighting and automotive applications or beam shaping. The main advantage of using HOEs is their high space bandwidth product and flatness. It is well known that the quality of the projected pattern is proportional to the amount of functional pixel in the hologram plane [1]. However, the achievable pixel pitch is limited by parameters in the fabrication process. This reduces the number of functional pixels per area, thus limiting the projection quality of the entire hologram.

In this paper, we present an approach to improve the projection quality for a given pixel pitch of the fabricated functional pixel by considering the individual shape of each functional pixel. We show that the consideration of the slope reduces the overall error in the reconstruction plane by a factor of eight compared to a pixel with the same size and a constant height.

2 Computer-generated hologram

A holographic optical element consists typically of one or more computer-generated holograms (CGH). For the calculation of a CGH numerical methods based on light propagation phenomena and constraints like intensity distributions are used. In this paper we use the Gerchberg-Saxton-algorithm (GSA) for the calculation of phase only holograms [2]. This algorithm allows the calculation of the complex wavefield in the hologram plane depending on the desired intensity distribution, projection distance, illumination wavelength, physical hologram size, and pixel pitch. The quality of the projection is determined by the space bandwidth product (SBP), given by the physical size of the hologram times the spatial frequency bandwidth represented by the pixel pitch of the functional pixel. Therefore, the quality is proportional to the number of functional pixels that can be manipulated [1,3]. To improve the quality of the reconstruction, the complex wavefield is calculated with a higher SBP, by numerically introducing subpixels in the area of a functional pixel. Therefore, in the numerical calculation the pixel pitch of the hologram is reduced and a so called high-resolution phase only hologram is calculated. The existing methods of down sampling the hologram produces a multitude of deviations and a loss of quality of the reconstructed intensity distribution. To overcome this limitation, the individual shape of each functional pixel is considered. The calculation of the shape coefficients takes place in a Taylor series expansion of the complex wavefield. The implementation of this Taylor series expansion of the complex wavefield takes place in the hologram plane in the GSA and is independent from the used propagation operator, e.g. Fresnel transformation or angular spectrum. The Taylor series expansion of the wavefield can be written as [4]

$$f(x) = \sum_{n=0}^{N} \frac{f^{(n)}(a)}{n!}(x-a) \tag{1}$$

Here, $f(x)$ is the complex wavefield within a functional pixel, $f^{(n)}(a)$ is the n-th derivative of f at the point a and $n!$ is the factorial of the Taylor order n. A scheme of the surface profiles of functional pixels for the Taylor series expansion with $N = 0, 1, 2$ is shown in Fig.1. The usually realized down sampling of the wavefield corresponds to the zeroth order $(N = 0)$ of the Taylor series. For $N = 1$ the Taylor series includes the zeroth and first order and corresponds to a tilted surface. If $N = 2$ the expansion includes the zeroth, first, and second order and corresponds to a parabolic surface.

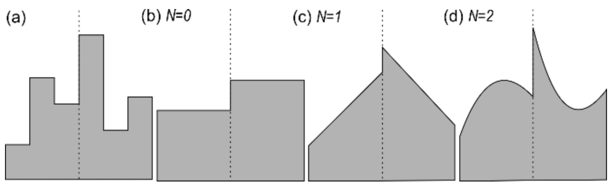

Fig. 1 *Scheme of the profile of two functional pixels with 3 subpixels, for (a) a high-resolution structure and the corresponding profiles to Taylor series expansions with N=0 (b), N=1 (c) and N=2 (d).*

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The expansion takes place for each functional pixel. In the numerical model each functional pixel will be approximated by a constant number of subpixels of the calculated high-resolution wavefield. Based on the necessary derivations in Eq. 1, is split into real and imaginary part and the Taylor series expansion will be executed separately from each other. Afterwards, the real and imaginary part is combined to a complex wavefield. The pixel shape depends on the regarded orders of the Taylor series expansion, see Fig. 1.

3 Influence of the pixel shape

Based on the deviations of the calculated high-resolution wavefield and the manufacturable structures, the pixel shape has a substantial influence on the projection quality. The consideration of inclined functional pixels enables an enhancement of the phase in the hologram plane compared to the usually down sampled phase of a high-resolution phase only hologram. The enhancement is the result of the additional degree of freedom by the individual pixel slope. In Fig.2(a) the phase of a calculated high-resolution wavefield with an approximation of 3x3 subpixels for one functional pixel is shown. In (b) and (c), the utilization of Taylor series expansion up to the zeroth order and the first order is shown, respectively. The first order expansion is in good agreement with the phase of the high-resolution wavefield.

Taking into account the individual slope of the functional pixels a reduction of the variance of the difference between the reconstruction of a high-resolution hologram and a hologram with individual pixel slopes up to a factor of 8 compared to hologram with the same pixel size and a constant value is demonstrated. The reduction of the variance indicates a reduction of the overall error in the reconstruction plane. Figure 3 shows the reconstruction of the simulated wavefields presented in Fig. 2. The reduction of the phase error in the hologram plane leads to a decreasing intensity in higher diffraction orders in the reconstruction plane. The intensity in the zeroth diffraction order increases by the same amount. Consequently, the reconstruction has a better contrast ratio compared to a down sampled hologram.

4 Conclusion

In this paper we presented an approach for the calculation of computer generated phase only holograms. We introduce an individual slope of each functional pixel by considering a new degree of freedom. This approach is based on the Taylor series expansion of a high-resolution wavefield, which is used for the numerical approximation of the slope. The reduction of the deviation of the phase distribution in the hologram plane reduces the overall error in the reconstruction plane by a factor of approximately 8 as compared to subsampled holograms with a constant pixel shape.

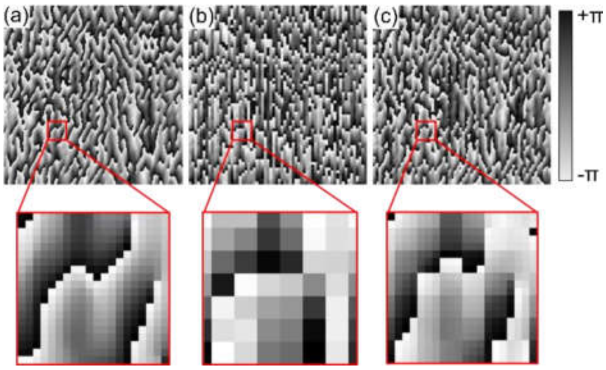

***Fig. 2** Phase of the calculated wavefield in the hologram plane with 3x3 subpixels for one functional pixel. Phase distributions of (a) the high-resolution wavefield, (b) of the zeroth order expansion, and (c) of the first order expansion.*

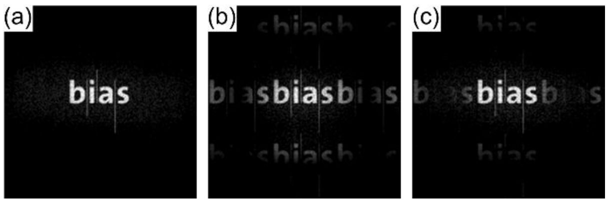

***Fig. 3** Numerical reconstructed intensity of phase only holograms from Fig.2. (a) high-resolution hologram, with an approximation 3x3 pixel for one functional pixel, (b) of the zeroth order expansion, and (c) of the first order expansion of the high-resolution hologram.*

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