

# Modelling the influences of technical surfaces on Phase Measuring Deflectometry

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In Phase Measuring Deflectometry (PMD) the camera is the most thoroughly modelled component, followed by the display with its nonlinearity and flatness deviation influencing the measurement uncertainty. However, the influences of the properties of technical surfaces on measurement deviation still lacks investigation. In this paper, a simulative approach for assessing these effects is presented.

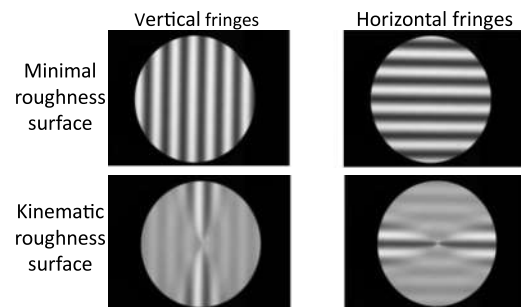
## 1 Introduction

PMD is commonly used to measure specular surfaces. The measured phase corresponds to the position on the display due to intersection of a vision ray [1] after being reflected from the specular surface for PMD modelling. A vision ray is a line of sight assigned to each camera pixel in the measurement volume using reverse ray tracing. For an ideal specular surface raytracing in the principle reflection direction is sufficient to obtain the fringe images whereby the intensity captured by each camera pixel corresponds to the intensity of the monitor position where the reflected vision ray intersects. Using the phase shifting method fringe images are converted to phase images and the absolute phase in vertical and horizontal direction is calculated. However, technical surfaces display both reflection and scattering due to the presence of microstructures. Technical surfaces manufactured using diamond turning are rotational symmetric around the center and display anisotropic scattering. Therefore, the vision ray from a camera pixel where the camera is focused on the surface will provide a scattered lobe from the surface compared to a single reflected vision ray from a specular surface.

## 2 Anisotropic scattering

In a previous investigation described in Ref. [2], the results of anisotropic scattering from technical surfaces using a plane surface with minimal roughness ( $R_a=4\text{ nm}$ ,  $R_z=44\text{ nm}$ ,  $R_q=5\text{ nm}$ ) and another surface with kinematic roughness ( $R_a=128\text{ nm}$ ,  $R_z=3.29\text{ }\mu\text{m}$ ,  $R_q=250\text{ nm}$ ) of same sizes are shown. For a white dot of  $15 \times 15$  monitor pixels moved on the monitor, the captured camera images of the reflected dot from the minimal roughness surface are circular and uniformly distributed with high contrast. But the presence of anisotropic scattering from the kinematic roughness surface results in lower contrast dots with different shapes and sizes along the tangential and radial directions depending on the position of reflection of the dot from the surface. The reflected dots also exhibit symmetric changes in

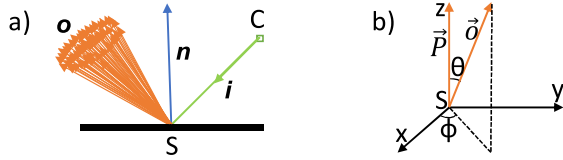
shapes and sizes around the centre due to the rotationally symmetric microstructures on the diamond turned surfaces. Fig. 1 shows fringe images resulting from both the surfaces when the fringe width on the monitor is chosen to be 50 monitor pixels. As shown in the Fig. 1 these fringe images which are distorted due to the kinematic roughness on the surface result in phase measurement deviations.



**Fig. 1** Fringe images due to the minimal roughness and kinematic roughness surface in vertical and horizontal directions. Due to the anisotropic scattering from the kinematic roughness surface the fringes parallel to the tangential direction result in distorted low contrast fringes and the fringes perpendicular to the tangential direction result in good contrast fringe images.

## 3 Reverse raytracing using BRDF model

To model the measurement process using inverse raytracing (vision rays) in PMD, every vision ray interacting with the technical surface results in a principal direction of reflection along with a bundle of scattered rays representing the scattered lobe. The intensity captured at any camera pixel will be the weighted average of intensities at these intersection points between the scattered rays and the monitor. The weights for the individual scattered rays can be assigned using the Bidirectional Reflectance Distribution Function (BRDF), defined as the ratio of reflected radiance in a given direction to the differential irradiance. Fig. 2 displays the modelling of the scattering from the technical surface.



**Fig. 2** a) Reverse raytracing for scattering from a technical surface. Here  $i$  is the incoming vision ray,  $o$  is the outgoing scattering ray bundle and  $n$  is the normal at the point of incidence on the surface. The intensity at the camera pixel  $C$  is calculated by the weighted average of the intensity at the intersection point between the individual rays from  $o$  and the monitor. b) Sampling of rays in scattered ray bundle in polar coordinates,  $\vec{P}$  is the ray in principal direction of reflection and  $\vec{o}$  is any individual ray direction in the scattered ray bundle.

Among the many existing BRDF models, the anisotropic scattering from diamond turned surfaces can be modelled suitably using the Ward BRDF model [3, 4]. When one vision ray from a camera pixel  $C$  is incident on the surface point  $S$ , then  $n$  is the normal at that point. At  $S$ , a local coordinate system can be defined where the z-axis coincides with  $n$ , x and y-axis are the tangential and radial directions, respectively. If  $\vec{i}$  is the vector from  $S$  to  $C$ ,  $\vec{n}$  is the normal vector at  $S$  and  $\vec{o}$  is one of the scattered rays' directions from  $S$  to monitor. The Ward BRDF model can then be described by

$$f(\vec{i}, \vec{o}) = \frac{\rho_s}{4\pi\alpha_x\alpha_y\sqrt{(\vec{i}\cdot\vec{n})(\vec{o}\cdot\vec{n})}} \times e^{-\left(\frac{(\frac{h_x}{\alpha_x})^2 + (\frac{h_y}{\alpha_y})^2}{(\vec{h}\cdot\vec{n})^2}\right)}$$

Here  $\vec{h} = \vec{i} + \vec{o}$ , the parameters  $\alpha_x$  and  $\alpha_y$  are responsible for the scattering width of the lobe in x and y axis, respectively.  $\rho_s$  is the parameter which controls the magnitude of the scattering lobe. If  $I_C$  is the intensity captured at the camera pixel  $C$  (from which the vision ray originated) and  $I_p$  is the intensity of the monitor pixel where the  $p$ th scattered ray out of  $N$  rays (including the ray  $\vec{P}$  in the principal direction of reflection) intersected, then

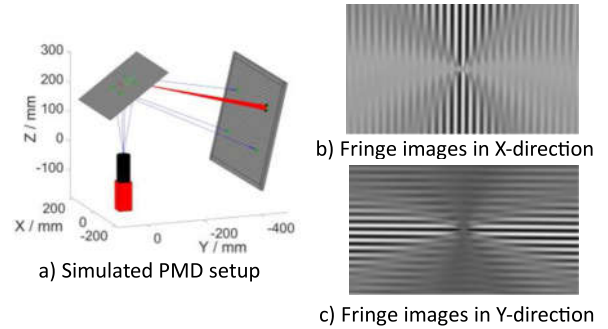
$$I_C = \frac{1}{N} \sum_{p=1}^N f_p \cdot I_p$$

and is thus given by the sum of the product of each individual ray weight (assigned by BRDF model) and the intensity at the position where the ray intersects the monitor.

However, the sampling of each ray in this scattering lobe has to be done uniformly as shown in Fig. 2(b). Hence a local coordinate system with origin at  $S$  is considered where the z-axis coincides with the ray at the principle reflected direction ( $\vec{P}$ ). Any scattered ray ( $\vec{o}$ ) in the lobe can be represented in a polar coordinate system with inclination angle  $\theta$  with the local z-axis and azimuth angle  $\phi$ . The maximum inclination angle  $\theta_{max}$  has to be selected such that the BRDF values for the rays with inclination angle greater than  $\theta_{max}$  are negligible.

## 4 Simulated results and conclusion

In the simulation all the surfaces are taken as flat surfaces with anisotropic scattering modelled on each surface point (where the vision ray from camera pixels meet the surface) using the parameters  $\alpha_x$ ,  $\alpha_y$  and  $\rho_s$ . Fig. 3 shows an exemplary simulated PMD setup with a flat plane and the resulting fringe images in both X, Y directions.



**Fig. 3** An exemplary simulated setup and resulting fringe images. a) PMD setup simulated using a flat rectangular surface (300 mm x 200 mm) with  $\alpha_x = 0.025$ ,  $\alpha_y = 0.002$ ,  $\rho_s = 0.4$ ,  $N = 100$  and  $\theta_{max} = 1^\circ$ . The rays highlighted in red are the scattered ray bundle from the vision ray originated from camera pixel (1,1). b) and c) shows the fringe images with width 50 monitor pixels in X- and Y-direction respectively. The simulated fringe images show the effect of anisotropic scattering from the surface similar to Fig. 1.

The simulation results demonstrate the validity of our approach for simulating rough surfaces by reverse raytracing. The BRDF model can generate fringe images similar to the experimental results. Future work has to establish the functional relationship between the surface properties and BRDF parameters to simulate the anisotropic scattering according to the surface properties.

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## References

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