Optische Systeme im Phasenraumbild

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Optical Microcavities are established model systems for non-linear dynamics. This motivates their description in phase space using Husimi-Functions, which give insight into the main radiation directions and the mode distribution at the dielectric interface. In this work, we generalize the application of Husimi-Functions to coupled microcavity systems.

1 Introduction

Optical microresonators have attracted growing interest due to their wide range of photonic and sensing applications [1]. In addition, the openness of the system, the interplay between shape and dynamics as the possibility to describe them using ray and wave models make them a great system for studying non-linear dynamics.

Tools from non-linear dynamics can be used to analyze the properties of optical microcavities from a new angle. Describing the dynamics in phase space, rather than real space, gives insight into all the properties at each unique point of phase-space. This representation comes with the drawback of having to reduce the dimensionality of the system in order to map it, but it is a small one since the interesting physics takes place at the dielectric interface. For resonant optical systems, the intensity is displayed at each boundary point (arc length s) and direction (p). The Poincarè map (ray optics) and Husimi functions (wave optics) are the most widely used phase space-representations in optical billiards. In this work, we apply Husimi functions to coupled optical systems and generalized boundaries.

2 Husimi Functions for Dielectric Cavities

The Husimi function is generally determined by the overlap of the wave function – in this case the electric field – with a coherent state of minimum uncertainty, a Gaussian wave packet [2]. Hentschel et. al. generalized Husimi functions for open dielectric systems [3], the result being an intuitive (quasi-)probability distribution in the reduced phase space at the boundary of the resonator. The four Husimi functions correspond respectively to the intensities for incident or emerging radiation inside and outside of the dielectric interface and have the following form:

$$H_j^{inc\ (em)}(s,p_j) = \frac{k_j}{2\pi}\left|(-1)^j F_j h_j(s,p_j) + (-)\frac{i}{k_0 F_j} h_j'(s,p_j)\right|^2, \tag{1}$$

where $j = 1(0)$ corresponds to the interior (exterior) of the resonator, s is the arc length and p the sine of the angle of incidence ϕ at the boundary. $F = \sqrt{n_j \cos(\phi)}$ is a weighting factor and $h_j^{(\prime)}$ the overlap of the electric field (it's normal derivative) with the gaussian packet ξ:

$$h_j(s,p) = \int_0^{2\pi} E(s')\xi(s';s,p)ds' \tag{2}$$

$$\xi(s';s,p) = c(\sigma)\sum_l \mathrm{e}^{\left(-\frac{1}{2\sigma}(s'+2\pi l-s)^2 - ik_0 p(s'+2\pi l)\right)} \tag{3}$$

σ controls the uncertainty in the s direction, $c(\sigma)$ is a normalization constant for the gaussian packet. In this work, the electric field of resonant modes has been calculated by solving the Helmholtz equation. Fig. 1 shows the outgoing Husimi function inside the cavity for the so-called quadrupole cavity as well as the mode distribution for this eigenmode.

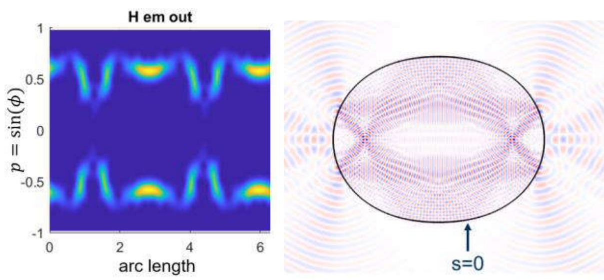

Fig. 1 *Outgoing Husimi function for the inside of a Quadrupole cavity. Electric field distribution of an eigenmode with kR =49, n=2.65. Note that the graphs are scale-free since they could be arbitrarily normalized.*

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3 Husimis for Coupled Systems

An alternative intuitive explanation for the calculation of the Husimi functions comes from its mathematical similarities with a windowed Fourier transformation, as can be readily seen from Eq (1-3). This allows us to compute the Husimi functions not only for the dielectric boundary but for any parametrized curve $s(t)$, given that the refractive index n is constant along the boundary and that the electric field is known. Since we are just mapping the angular and intensity components to each point in phase space our operation is not restricted to any physical boundary. This method does not facilitate the study of single resonator systems, since the most useful phase-space description is the one at the boundary. It is however a powerful tool to analyze the dynamics in systems of coupled resonators by looking at arbitrary boundaries and thus obtaining additional information about the coupling and its effects.

This method has been applied to explore the coupling effects in a chain of Limaçon cavities (following the results in [4]). Husimi functions could be used to reveal exact coupling positions and angles which could not be seen in real space due to the interaction between mode and coupling. In addition, by looking at Husimis on boundaries outside the resonator it was possible to extract additional information of the system for varying coupling distances and resonator positions (e.g. comparision of an edge resonator with a central resonator). Due to space limitations, we focus here mainly on a second example involving a shortegg resonator coupled to a waveguide.

4 Shortegg Cavity and Waveguide

The so-called shortegg cavity (Fig 3. d) displays a directional far-field pattern for a refractive index as low as $n = 1.5$, showing potential for photonic applications. Due to the lack of rotational symmetry, the transmission curves show a strong dependence on the angle between the waveguide and the cavity orientation [5]. In order to explain this phenomenon it is useful to look at the incoming Husimi function for the cavity mode (Fig. 3a) and compare it to the Husimi function of the waveguide excitation (Fig. 3b), which can be done by simulating only the waveguide and using the field distribution along the boundary where the resonator should be. This allows us to identify coupling positions as well as angles, giving more information than an analysis relying only on real space. Depending on the angle between waveguide and cavity, these two Husimi functions will shift in respect to each other. The coupling efficiency can be directly correlated to the overlap of the Husimi functions.

A Husimi function of the mode excited via a waveguide (note that Fig. 3d shows only the eigenmode without a waveguide) is shown in Fig. 3c and

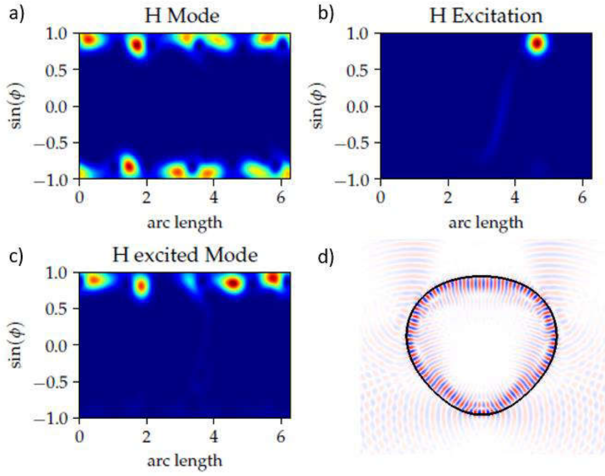

Fig. 2 *Husimi functions for the shortegg cavity coupled to a waveguide. (a) Husimi for the eigenmode corresponding to (d). (b) Husimi along the same boundary for excitation only in the waveguide. (c) Husimi of the excited mode. (d) Field distribution of the analyzed eigenfrequency (without waveguide).*

displays the expected distribution patterns. Excited waves travel only in one direction, leading to vanishing Husimi-values for incident angles in the other direction. In addition, one can see and quantify the change of the intensity distribution along the boundary due to the waveguide excitation compared to the eigenmode.

5 Conclusions

A phase space representation of the wave dynamics in microcavities has been presented and used to study coupling effects in open optical systems. The angle dependence of the coupling intensity for a waveguide-resonator system has been explained using the Husimi functions. In addition, the method can provide further insight into the coupling mechanisms at arbitrary boundaries in-between cavities.

References

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