Étendue Estimation for Non-Trivial Geometry

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The light distribution of a source, in an optical system or at a target is described in a 4D phase space of the position and direction of rays [1]. Étendue is volume in this phase space, and its knowledge allows for calculating the achievable flux.
Sometimes, it is easy to analytically calculate the étendue of a certain configuration. In other cases, this is impossible or requires considerable effort. We show that in such cases, alternative ways of calculation or simulations can help.

1 Introduction

The highest possible flux that can be collected by a target of étendue U_T is

$$\Phi_T = \min(\Phi_S, U_T \cdot L_S) \tag{1}$$

from a source with flux Φ_S and (homogeneous) luminance L_S.

Usually, the target étendue is defined by the customer—sometimes not by its value, but by describing a situation. The first task in illumination design is to calculate the target étendue for an estimate of collectable flux in the pre-design phase of a project.

Étendue is defined by [2, 3]

$$U = \iint n^2 (\cos\vartheta)^2 \, \mathrm{d}\Omega \, \mathrm{d}A \approx A \cdot \Omega \tag{2}$$

the latter approximation being valid for the paraxial case in air ($n = 1$) [4].

The same way, two disks (radii r_1, r_2) at a large distance $d \gg r_i$ define an étendue of

$$U = \left(\frac{\pi \, r_1 \, r_2}{d}\right)^2 \tag{3}$$

Special effort is necessary in more complex situations, as in Fig. 1.

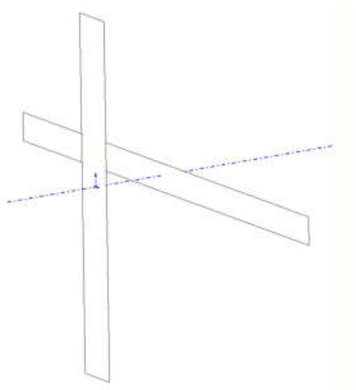

Fig. 1 *Étendue defined by crossed slits*

Another example is an (especially off-axis) aperture between source and target (Fig. 2). Finally, we may consider an optical system between source and target, such as in a DLP or LCD projector.

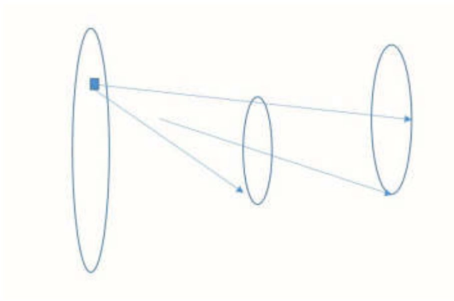

Fig. 2 *Étendue defined by* three *apertures*

2 Analytic Solutions

The case of two *close* disks ($d \approx r_i$) can be treated with some assistance from 2D optics [5, 6, 2] :

$$U = \frac{\pi^2}{4}\left(\sqrt{(r_1 + r_2)^2 + d^2} - \sqrt{(r_1 - r_2)^2 + d^2}\right)^2 \tag{4}$$

It can be used to estimate geometric losses of an LED coupled to a light guide (Fig. 3).

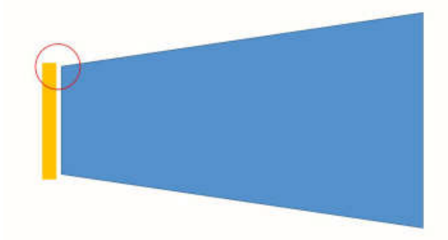

Fig. 3 *LED coupled to a light guide. The width of the air gap causes geometrical losses.*

If one or both disks are replaced by rectangular areas, an approximate solution can be obtained just by including the area ratio (rectangle to disk). There are exact solutions possible with some mathematical effort. We refer

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to extensive knowledge in the thermal radiation community (`www.thermalradiation.net/tablecon.html`). Their wording may be different to ours (*configuration factor* or *view factor* instead of étendue), but the content is relevant for our task [7, 8].

One may even ask how much light is absorbed by the source itself (the *self-irradiation co-efficient*), important for concave sources or (in the past) for incandescent filaments. One extreme example is described in [9].

3 Étendue Measurement by Simulation

Instead of solving the problem analytically, for an arbitrary geometry we may use our preferred non-sequential software, using the fact that luminance is density in phase space, $L = \mathrm{d}\Phi / \mathrm{d}U$. Thus, when luminance is constant, étendue and flux are proportional.

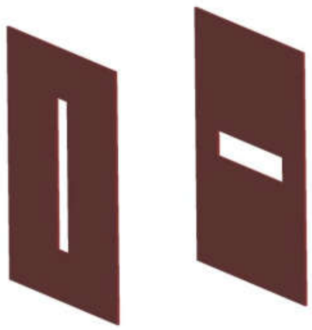

Fig. 4 *Étendue defined by crossed slits (as of Fig. 1) in a simulation*

The procedure is:

- Set up the scenery
  - building any area of the étendue definition as an aperture (such as in Fig. 4)
  - set up any other optical elements (such as lenses or homogenizers) between source and target
  - ensure no absorption, no partial reflection, no ray splitting and no scattering
  - place a Lambertian source in front of the system (it is useful to give it the size of the first aperture)
  - place a detector after the system
- Run the simulation
- Measure the detector flux
- Calculate target étendue U_T as follows:

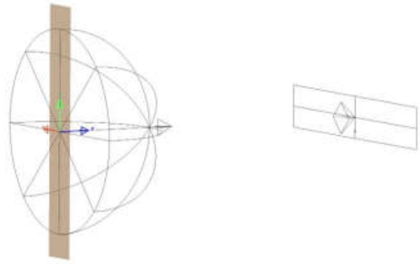

Fig. 5 *Slits represented by source and target area*

The luminance of a Lambertian source is given by

$$L_S = \frac{\Phi_S}{A_S \, \pi \, \mathrm{sr}} \tag{5}$$

Étendue from source to target is calculated from target flux and source luminance

$$U_T = \frac{\Phi_T}{L_S} = \frac{\Phi_T}{\Phi_S} \cdot A_S \, \pi \, \mathrm{sr} \tag{6}$$

This procedure yields the system étendue as a whole (there may be much more information hidden in the details). It works with an optical system between source and target, too.

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