7D-Metrology: The coherence function as a solution to complex problems of optical metrology

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The coherence function offers new possibilities for realizing robust, precise and fast measurements in complex measurement situations. In the most general case, it maps a 7-dimensional space onto a complex-valued function. The challenge now lies in researching efficient processes and techniques for its use, e.g. for shape measurement.

1 Introduction

Conventional interferometric measurement methods determine the phase or complex amplitude of a quasi-monochromatic wave field. In contrast, the determination of the coherence function of light considerably expands the possibilities of interferometric measurement technology. Since this function in the most general case maps a 7-dimensional (7D) space to a complex-valued function, new challenges arise in the research and development of efficient processes and techniques for using the coherence function, e.g. for shape measurement. For this purpose, measurement techniques for recording the coherence function and efficient evaluation strategies are required.

2 From the wave function to the coherence function

Assuming a quasi-monochromatic, time dependent optical wave field $U(\vec{x}, t)$ is given in the input plane of a measurement system. The coherence function $\Gamma(\vec{x}_1, \vec{x}_2, \tau)$ describes the covariance of the wave field at different locations and times given by

$$\Gamma(\vec{x}_1, \vec{x}_2, \tau) = \langle U^*(\vec{x}_1, t) U(\vec{x}_2, t + \tau) \rangle_T \qquad (1)$$

with the time average $< ... >_T$ of the product of $U(\vec{x}, t)$ at two different locations and a variable time shift τ. We therefore obtain an alternative description of light depending on a 7D-Space (6D spacial dimensions and one time dimension).

Figure 1 schematically shows the most general case of the measurement problem and the dimensions involved using Computational Shear Interferometry (CoSI) und Γ-Profilometry. Both techniques will be described in the following paragraph.

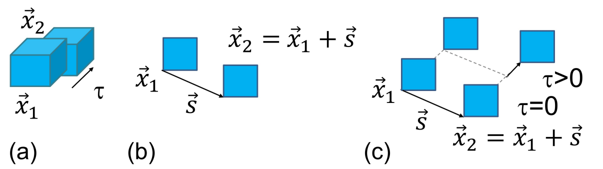

Fig. 1 *Dimensionality of the measurement problem for measuring the Γ-function and selected measurement approaches. a) General case (7D): Two 3D-spaces and a time shift τ. b) CoSI (4D): Two planes mutually separated by a variable shear $\vec{s}$, $\tau = 0$. c) Γ-Profilometry (5D): Two planes with variable shear $\vec{s}$ and mutual shift in time τ.*

3 Measurement techniques for sampling the coherence function

3.1 Computational Shear Interferometry (CoSI)

Given the complex time-independent amplitude $u(\vec{x})$ of an optical wavefield, a shear interferometer measures the coherence function

$$\Gamma(\vec{x}, \vec{x} + \vec{s}) = u^*(\vec{x}) \cdot u(\vec{x} + \vec{s}). \qquad (2)$$

at $\vec{x}$ und $\vec{x} + \vec{s}$. Ideally, the mutual time delay τ of the wave fields is constant or zero and thus neglected. The determination of the wave field is obtained by minimizing the functional

$$L(f) = \sum_{n=1}^{N} \| \Gamma_n(\vec{x}) - f^*(\vec{x}) f(\vec{x} + \vec{s}_n) \|^2. \qquad (3)$$

The index n distinguishes several measurements necessary to obtain a unique solution. The function $f_{\text{min}}(\vec{x})$, that minimizes L is the best approximation for $u(\vec{x})$ [1]. An extension of this approach allows the simultaneous use of several light sources [2].

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3.2 Γ-Profilometry

Due to the surface profile of a sample, light reflected from parts of the sample with a difference in height Δh has to travel an additional distance of $2\Delta h$ to travel back to the sensor and consequently requires an additional time $\tau = 2\Delta h/c$. Since the coherence function Eq. 1 depends on the time delay τ, it allows for the detection of the surface profile by investigating shifts along the time axis.

The spectral bandwidth of a typical LED is $\nu_B = 13\,$THz. The coherence time is given by

$$\tau_c = \frac{4\ln 2}{\pi} \cdot \frac{1}{\nu_B}. \quad (4)$$

and thus results in 67 fs corresponding to a coherence length of $l_c \approx 20\,\mu$m.

With a limited τ_c, $|\Gamma(\tau)| > 0$ only holds for a small interval along the τ-axis. We can therefore detect the shift τ by sampling of Γ across the temporal domain. Combining the temporal sampling with spatial sampling at numerous positions $\vec{x}_m$ and $\vec{x}_m + \vec{s}$ using a shear interferometer as described above, the complete surface profile is accessible [3].

4 Methods for efficient data collection and evaluation

One of the main challenges in exploiting the full potential of the coherence function is the huge amount of sampling required to properly record it. Here, we identify two potential candidates to solve this problem.

4.1 Compressed Sensing (CS)

According to the Shannon-Nyquist theorem, a lot of data are required to fully capture signals as per their spectral bandwidth. Obvious symmetry structures can be exploited to reduce sampling, but non-obvious structures interconnected with object properties are harder to exploit. Compressed Sensing (CS) allows to efficiently sample signals with inherent structures that can be described by a few non-zeros coefficients in a known (e.g. Fourier) base or a learned dictionary [4] and thus allow for strong sub-sampling. Dictionary Learning offers tools to learn such sparse descriptions of signals [5]. Furthermore, CS provides different approaches how sub-sampling can be integrated into the optical setup before analog-to-digital conversion thus reducing the number of samples to be processed [6].

4.2 Machine Learning (ML)

Extracting interpretable information from 7D data can be achieved by using methods from pattern recognition [7] or data visualization [8]. Once parameter correlations have been learned, it is even possible to perform process parameter optimization on the measurement technique itself [9].

5 Conclusions and Outlook

The coherence function enables new strategies for optical metrology. Techniques based on computational shear interferometry in combination with compressed sensing and machine learning may open a path to an efficient use of this novel approach.

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